CURRICULUM FOR
MSc PROGRAMME IN MATHEMATICS
at RAMAKRISHNA MISSION
VIVEKANANDA EDUCATIONAL AND RESEARCH INSTTITUTE

Course No.	***Semester 1***	***Course No.***	***Semester 2***
1)	Real Analysis 1	5)	Measure Theory
2)	Algebra 1	6)	Algebra 2
3)	Topology	7)	Algebraic Topology
4)	Linear Algebra1		Linear Algebra2
5)	Complex Analysis		Real Analysis 2
Course No.	***Semester 3***	***Course No.***	***Semester 4***
9)	Functional Analysis	13)	Number Theory
10)	Differential Geometry	14)	Elective 3
11)	Elective 1	15)	Elective 4
12)	Elective 2	16)	Project

Notes: 1) Under special circumstances, courses may be exchanged between semesters 3 and 4 in order, for instance, to accomodate distinguished visitors who might be willing to offer courses.

2) Each semester has 4 courses of 4 credits each, except for the last semester where there are 3 courses of 4 credits each and one project of 12 credits. This gives rise to a total of 72 credits for the programme.. The MSc course is 4-semester long. At the end of each semester, a letter grade will be assigned for each course. These will carry weights as follows: A-10, B-8, C-6, D-4, F-0. The average of the grade-points obtained in courses weighted by credits and normalized out of 10.0 will be called the *Semester Performance Index (SPI).* The cumulative average of the SPI's is the *Cumulative Performance Index (CPI).* Grading will in general be relative with an absolute pass-mark of 30%.

3) One credit corresponds to roughly one work hour per week. Each semester will typically be of 15-16 weeks in duration.

COMPULSORY COURSES

FIRST YEAR: SEMESTERS 1, 2

SEMESTER 1

M 201: Algebra 1

Syllabus:

Group Theory
Group action on a Set, Stabilisers and kernels, Cayley's theorem, Class Equation, Automorphisms, Sylow's Theorem, Direct Products,

If time permits, Semidirect Products.

RingTheory
Examples, Ring Homomorphisms, Ideals, Quotient Rings with emphasis on Z/nZ and Modular Arithmetic, Isomorphism Theorems, Properties of Ideals, Prime and Maximal Ideals, Rings of Fractions, Chinese remainder theorem;
Euclidean domains, PID, UFD, Factorization in Z[i].
Polynomial Rings: Definition and basic properties, division algorithm; R[x] is a UFD if R is a UFD; Irreducibility Criteria; Noetherian rings.

If time permits,
Hilbert basis theorem and applications;
Definitions and examples of Algebraic Sets and Coordinate rings. Polynomial Maps.

Suggested Text:
D.S. Dummit and R.M. Foote - Abstract Algebra J. Wiley
Other texts:
M. Artin - Algebra - PHI
I. N. Herstein - Topics in Algebra; J. Wiley
N. Jacobson - Basic Algebra Ch. 1,2. Van Nostrand

M 202: Topology
Syllabus:
Topological Spaces and Continuous Functions: Basis, Order and Product Topology, Closed Sets and Limit Points;
Metric Topology, Completion of Metric Spaces, Baire Category Theorem;
Product Topology;
Connectedness and Compactness: Connectedness and local connectedness, compactness and local compactness;
Separation and countability axioms, T1, T2, T3, T4, Urysohn Lemma, Tietze extension theorem;
Tychonoff theorem.

Quotient topology and identification spaces; Topological manifolds as examples of quotient topology - torus, Klein's bottle, projective spaces;
Examples of topological groups;
Classification of 1 and 2-manifolds (statement and illustration only);
Homotopy of paths, Fundamental Group; Covering spaces and group actions on spaces,

computation of fundamental group of the circle. Fundamental groups of surfaces.

Suggested Texts:
J.R. Munkres - Topology Ch. 2 (sec 12-20), 3 (sec 23-29), 4(sec. 30-35), 5(sec. 37)
sec 22 of Ch 2 for quotient topology, section on topological groups, Ch. 9, sec 51-55, 58-60 for Fundamental Group); PHI
G.F. Simmons - Topology and Modern Analysis (ch. 2 sec. 9-13 for metric spaces); TMH
M. A. Armstrong - Basic Topology; Springer

Other texts:
I. Singer and J. Thorpe - Notes on Elementary Topology and Differential Geometry. Springer
P. J. Higgins – An Introduction to Topological Groups – LMS lecture notes CUP
J.W. Milnor - Topology from a differentiable viewpoint (for notion of manifolds and classification of 1 and 2-manifolds) PUP
S.M. Srivastava - A Course on Borel Sets, Springer

M 203: Complex Analysis
Syllabus:
Analytic Functions: Power series, Trigonometric functions, Cauchy - Riemann equations, analytic functions as mappings;
Complex integration: Cauchy's theorem and integral formula, power series representation, zeros of an analytic function,
Meromorphic functions and residue calculus, Index of a closed curve, Morera's theorem, Liouville's theorem, open mapping theorem;
Singularities: Classification, Rouche's theorem, argument principle;
Maximum modulus principle, Schwarz lemma, analytic continuation;
Compactness and convergence in the space of analytic functions: Space of continuous functions, space of analytic functions, normal families, space of meromorphic functions, Riemann mapping theorem

Suggested text:
J.B. Conway - Functions of One Complex Variable: Narosa

Other texts:
T.W. Gamelin - Complex Analysis, Springer
L. V. Ahlfors - Complex Analysis, TMH
W. Rudin - Real and Complex Analysis TMH
S. Ponnasamy- Complex Analysis TMH
D.E. Sarason - Complex Function Theory HBA

Course M 204a: Linear Algebra 1

Syllabus:
Quotients of vector spaces, Geometric significance of cosets and quotient spaces, Dimension of Quotients of Vector Space;
Linear Transformations: Kernel, Image, Isomorphisms, rank and nullity, linear functionals, annihilators, Dual and Double dual, Transpose of a Linear Transformation.
Eigenvectors of a linear transformation.
Inner Product Spaces, Hermitian, Unitary and Normal Transformations, Spectral Theorems, Bilinear and Quadratic Forms.

Note: 1) It is desirable to hold problem sessions, so that the students gaina firm conceptual grasp as well as the capacity to solve problems. The initial sessions are to review basic topics like elementary matrices and elementary operations, invertible and elementary matrices, matrix concepts like similarity and rank and their relation to linear transformations.

2) For the topic *Eigenspaces*, it suffices to cover basic concepts necessary for Spectral theory. Minimal and characteristic polynomials, canonical forms will be done in Linear Algebra 2.

3) Geometric significance of concepts to be emphasized wherever necessary, e.g. geometric interpretation of specific linear operators, orthogonal matrices, determinant as volume, etc.

Suggested Text:
K. Hoffman and R. Kunze – Linear Algebra, PHI, Ch. 3 (with revision of 1), 6.1-6.2, 8, 9, 10.
Other texts:
D.S. Dummit and R.M. Foote - Abstract Algebra, John Wiley - Ch. 11
M. Artin - Algebra, PHI – Ch. 4, 7.
N. S. Gopalakrishnan – University Algebra, Wiley Eastern, Ch. 3.5, 5.1-5.7, 5.11-5.13.
I. N. Herstein – Topics in Algebra, John Wiley, Ch. 4.1-4.4, 6.1-6.3, 6.8-6.11.

M 205 a: Real Analysis
Syllabus:

Sequences and series of functions *(2 weeks):*
Uniform convergence, Equicontinuous families and Weierstrass Theorem, Ascoli theorem;

Suggested Texts:
W. Rudin - Principles of Mathematical Analysis - *Ch 7 Tata McGraw Hill*
Other texts:
T. M. Apostol - Mathematical Analysis - Narosa

Multivariable Calculus(5 weeks):
Functions of several variables: Differentiation, Contraction Principle, Inverse Function Theorem, Implicit Function Theorem; Rank Theorem; Jacobians, Differentiation of Integrals);

Suggested Texts:
T. M. Apostol - Mathematical Analysis - Ch. 12, 13, Narosa
W. Rudin - Principles of Mathematical Analysis - Ch 9, Tata McGraw Hill
Other texts:
M. Spivak - Calculus on Manifolds, Publish or Perish

SEMESTER 2

M 205 b: Real Analysis
Ordinary Differential Equations (3 weeks):
Ordinary Differential Equations: Cauchy-Peano Existence Theorem, Uniqueness, Picard-Lindelof Theorem; Continuation of solutions, systems of differential equations;

Suggested Text:
E. A. Coddington and N. Levinson - Theory of Ordinary Differential Equations (1.1-1.6), PHI

Partial Differential Equations (4 weeks):
Laplace equation: Mean Value Properties, Maximum and Minimum Principles, Harnack Inequality, Poisson Integral, Convergence Theorems, Dirichlet Problem.
If time permits:
Heat equation: fundamental solution.

Suggested Texts:
D. Gilbarg and N. Trudinger – Elliptic Differential Equations of Second Order, Springer
L.C. Evans, Partial Differential Equations, GTM v 19. AMS 1998

M 208: Measure Theory
Syllabus:
Sigma algebras of sets, Measurable functions;
Measures, outer measures, Extensions of measures, integration with respect to measure;
Examples of measures on intervals and the real line; regularity of measures.
Monotone Convergence Theorem, Fatou's Lemma, Dominated Convergence Theorem;
Examples from Lp spaces, Luzin's theorem, density of compactly supported smooth functions;

Radon-Nikodym theorem, absolutely continuous and singular measures;
Product of two measures and Fubini's theorem,,

Suggested texts:
G. de Barra - Measure theory and Integration (ch. 4-8, 10); Wiley
W. Rudin - Real and Complex Analysis (Ch. 1, 2, 3, 6) TMH

Other texts:
H.L. Royden - Real Analysis (Ch. 11-13); Prentice Hall
K. R. Parthasarathy – Measure and Integration, Macmillan, India
P.R. Halmos - Measure theory; Narosa
S. K. Berberian – Measure and Integration, Springer
M. F. Munroe – Introduction to Measure and Integration, AP
S. K. Berberian – Measure and Integration, Springer

M 206: Algebra 2 -- Field Theory

Syllabus:
Finite groups, simple groups, solvable groups, simplicity of A_n.

Field Theory: Algebraic Extensions, Finite and algebraic extensions, algebraic closure, separable and inseparable extensions, finite fields;
Galois theory: Galois extensions and Galois group, fundamental theorem;
Explicit examples and concrete applications of Galois theory;
Roots of unity, cyclotomic polynomials and extensions, solvability by radicals.
If time permits:
Introduction to Transcendental Extensions: Finite transcendence degree.
Integral Extensions;
Hilbert's Nullstellensatz, affine varieties.

Note: The topic on *finite groups* should be done just before the topic *Solvability by radicals.* Section 4.6 of Jacobson's Basic Algebra 1, or Section 5.7 of Herstein should be used.

Suggested text:
D.S. Dummit and R.M. Foote -Abstract Algebra, Wiley - Ch. 13, 14, 15.1 – 15.3
N.S.Gopalakrishnan - University Algebra, Wiley Eastern – Ch. 4
TIFR pamphlet on Galois theory.
Other texts:
S. Lang – Algebra, Addison Wesley, (Ch. 5, 6.1 – 6.7, 7.1, 8.1, 9.1);
I. N. Herstein – Topics in Algebra, John Wiley, Ch. 5.
N. Jacobson - Basic Algebra 1, HBA, Ch. 4
G. Rotman – Galois Theory, Springer

M 207: Algebraic Topology

Syllabus:

Singular homology and Eilenberg-Steenrod Axioms: Relative Homology, excision and exactness. Mayer-Vietois sequence, homotopy invariance;

Cellular homology as an example of a homology theory: Computation of homology for cell-complexes like S^n, CP^n, closed 2-manifolds.

Singular cohomology, cup and cap products, Cohomology ring;

Poincare duality for closed manifolds.

Suggested text:

A. Hatcher – Algebraic Topology (Ch. 2,3)

References:

M.J. Greenberg - Lectures on algebraic topology, Addison Wesley,

J. Vicks – Homology Theory, Springer;

J. R. Munkres - Elements of Algebraic Topology, Addison Wesley

Singer and Thorpe – Notes on Elementary Topology and Differential Geometry, Springer

Course M 204b: Linear Algebra 2

Syllabus:

Modules

Modules over Commutative Rings and submodules.

Examples: Vector Spaces; Abelian Groups, Commutative Rings; Ideals and Quotients, Invariant subspaces of a K-linear transformation of a vector space V as a K[X] submodule of V;

Module Homomorphisms, Kernel and Image, Hom(M,N),

Generation of modules, Direct sum and Free Modules.

Noetherian modules, Annihilator and torsion submodules;

Finitely generated modules over PID, submodule of a free module is free;

Structure theorems – Invariant factor form and elementary divisor form;

Primary decomposition theorems, (proof of uniqueness may be omitted) Application to abelian groups.

Introduction to Canonical Forms: Statements and Applications; *Outline of Proofs to be given. Details of Proofs may be excluded from the examination syllabus.*

Minimal and characteristic polynomials; triangularisation over algebraically closed field;

Cayley-Hamilton Theorem, Nilpotent transformations.

Rational and Jordan canonical forms.

Suggested Texts:

D.S. Dummit and R.M. Foote -Abstract Algebra J. Wiley

N.S.Gopalakrishnan - University Algebra Oxonian Press

K. Hoffman and R. Kunze – Linear Algebra, PHI (Ch 3.7- 3.10, Ch 8-10)
M. Artin - Algebra, PHI

Other texts:
I. N. Herstein – Topics in Algebra, John Wiley, Ch. 6.4-6.7.
N. Jacobson - Basic Algebra 1, HBA, Ch. 3

SECOND YEAR: SEMESTERS 3,4

SEMESTER 3

M 211: Functional Analysis
Syllabus:
Normed Linear Spaces and Banach Spaces: Bounded linear operators, Duals, Hahn-Banach theorem; Uniform boundedness principle;
Open mapping and Closed Graph theorems, some applications;
Dual spaces: Computing duals of L^p (1 < = p < infty) and C[0,1]; reflexive spaces;
Weak and weak* topologies, Banach Alaoglu theorem.
Hilbert Spaces - Orthogonal sets, Projection theorem, Riesz representation theorem, Adjoint operator; Self-adjoint, normal and unitary operators, Projections.
Spectrum and spectral radius; Spectral theorem for compact operators.
If time permits,
Spectral theorem for self-adjoint, normal and unitary operators;

Suggested Text:
G. F. Simmons - Topology and Modern Analysis (Ch. 9, 10, 11, 12), TMH
J. B. Conway - A First Course in Functional Analysis, Springer
Other texts:
W. Rudin – Real and Complex analysis TMH

M 212: Differential Geometry
Syllabus:
Manifolds: Smooth functions, vector fields, Jacobian, integral curves, submanifolds;
Connections and curvature for surfaces in R^3, Gauss map.

The classical theory of surfaces in R^3 to be stressed and done in detail as the first set of examples where the notions of connection and curvature come up. The general theory below to be described in the context of Riemannian manifolds only.

Riemannian manifolds and submanifolds: Length and distance, Riemannian connection and curvature, curves, submanifolds, hypersurfaces.

Operators on forms and integration: Exterior derivative, contraction, Lie derivative, general covariant derivative, integration of forms and Stokes' theorem;

Surfaces in R^3, Gauss-Bonnet formula and Index theorem.

Suggested texts:
N.J. Hicks - Notes on Differential Geometry; Ch. 1, 2, 3, 7, 8.1, 8.2, AP
Ch 5,6 of the above reference deal with the theory of connections and curvature in great detail and can be used as a reference for these topics, rather than a text. For this topic it is advisable to use Ch 2 and 4 of the text below as the basic text.
M. P. do Carmo - Riemannian Geometry (Ch. 1,2,3,4) Birkhauser

Other texts:
S. Kumaresan, A Course on Differential Geometry and Lie Groups, HBA
N.J. Hicks - Notes on Differential Geometry; Ch. 5, 6 AP
B. O'Neill - Elementary Differential Geometry; Springer
Klingenberg - Elementary Differential Geometry; AP
M. P. do Carmo - Differential geometry of curves and surfaces Birkhauser
M. Spivak - Calculus on Manifolds Publish or Perish
Singer and Thorpe – Notes on Elementary Topology and Differential Geometry Springer

Course 11: Elective 1
Course 12: Elective 2

SEMESTER 4

M 213: Number Theory
Syllabus:
Revision of Unique Factorization, Congruences, Chinese remainder theorem;
Structure of U(Z/nZ), quadratic reciprocity, Quadratic Gauss sums, Finite fields, Gauss and Jacobi sums, Cubic and biquadratic reciprocity, Equations over finite fields, Zeta Function.
Algebraic number fields and their ring of integers, Units and primes, factorisation, Quadratic and cyclotomic fields, Dirichlet L function, Diophantine equations, Elliptic curves.

Suggested texts:
Ireland & Rosen - A Classical Introduction to Modern Number Theory
I. Niven, S.H. Zuckerman and L.H. Montgomery - An introduction to the theory of numbers

Other texts :
J.P. Serre - A course in arithmetic,
J.W.S. Cassels and A. Frohlich - Algebraic Number Theory

Course 14: Elective 3
Course 15: Elective 4

Course 16: Elective 5/Project

ELECTIVES

Elective M 301: Advanced Complex Analysis

Syllabus:

Revision of Compactness and convergence in the space of analytic functions and Riemann mapping theorem;

Weierstrass Factorisation theorem, Riemann zeta function, Runge's theorem, Mittag-Leffler's theorem, Analytic continuation and Riemann surfaces, Schwarz reflection principle, Monodromy theorem, Harmonic functions, subharmonic and superharmonic functions, Dirichlet problem, Green's functions, Jensen's formula.

If time permits,

Hadamard factorisation theorem, Bloch's theorem, Picard's theorem

Suggested texts:

J. B. Conway - Functions of one Complex Variable (Ch. 7-12) Narosa

L.V. Ahlfors - Complex Analysis TMH

H.M. Farkas and I. Kra - Riemann surfaces, Springer

E. Stein and R. Shakarchi – Complex Analysis, PUP

Elective M 302: Harmonic Analysis

Syllabus:

Topological groups, quotients and products, open subgroups, Haar Measure on Locally Compact Groups; Properties of Haar measure, Invariant measures on homogeneous spaces.

Representation of compact Lie groups, Schur's Lemma, Weyl character formula, Peter-Weyl theory, Representations of SU(2,C).

If time permits,

Induced representation and Frobenius reciprocity theorem, Principal series representations of Sl(2,R)

Suggested texts: G.B. Folland: Introduction to Abstract Harmonic Analysis, CRC Press

S.C. Bagchi, S. Madan, A. Sitaram and U.B. Tewari - A first course on representation theory and linear Lie groups, University Press.

A. Deitmar - A first course in harmonic analysis, Springer

E. Stein and R. Shakarchi – Fourier Analysis, PUP

Elective M 303: Probability Theory

Syllabus:

Independence, Kolmogorov zero-one law, Kolmogorov three-series theorem, Strong law of large numbers, Levy-Cramer continuity, Central limit theorem, Infinite products of probability measures, Discrete-time discrete state Markov chains.

Suggested texts: J. Nevue - Mathematical foundations of the calculus of probability

P. Billingsley - Probability and measure

Y.S. Chow and H. Teicher - Probability theory, Independence, interchangeability, martingales.

Elective M 304: Distribution Theory

Syllabus:

C^\infty functions on R^n, smooth partition of unity on R^n;

Test function space on an open subset \Omega of R^n;

Space of distributions on \Omega, functions and measures as distributions;

Examples of distributions on \Omega that do not extend to distributions on R^n;

Elementary operations on the space of distributions: Derivatives of distributions, multiplication by a function, convolution by a test function;

Sequences of distributions: convergence and approximation by test functions.

Schwartz space, Isomorphism of Schwartz space with itself under Fourier transform;

Fourier inversion and Fourier-Plancherel Theorem;

Tempered distributions, Fourier transforms of tempered distributions;

Distributions of compact support;

Convolution of a tempered distribution with a function of Schwartz class;

Fourier transform of derivatives and convolutions (with a Schwartz class function) of tempered distributions;

Application of distributions to solving PDE's: Weak solutions, some easy examples, statement of elliptic regularity;

solution of Laplace equation on the half-plane and the Heat equation in R^3, using Fourier transforms.

Suggested Texts:

W. Rudin – Functional Analysis, TMH, Ch. 6, 7.1 – 7.19

R. Strichartz – A guide to distribution theory and Fourier transforms, CRC Press, 1994, Ch. 5

G.B. Folland – Fourier Analysis and its applications, Wadsworth and Brooks

S. Kesavan: Functional analysis and applications, John Wiley, 1989

Elective M 305: Operator Algebras

Multiplicative functionals and maximal ideal space, Gelfand transform, Gelfand Naimark theorem, Rational functional Calculus

C* Algebras,

Positive Cones of C* Algebras

States and GNS Construction

(If time permits, some of the following topics in C algebras may be touched upon*

Approximate Identitites, Extreme points on the unit ball,

Pure States and regular maximal ideals

Ideals, Quotients and Representations)

Banach space of operators on a Hilbert space B(H)
Locally convex Topologies on B(H)
Polar decomposition and orthogonal decomposition
von Neumann Double Commutant Theorem
If time permits,
Kaplansky's density theorem

Suggested Texts:
R. V. Kadison and J. R. Ringrose – Fundamentals of the Theory of Operator Algebras, AMS

Other texts:
Introduction to Operator Algebras – Li Bing Ren, World Scientific
W. Arveson – An Invitation to C* Algebras (Ch 1), Springer
V. S. Sunder - An Invitation to von Neumann Algebras (Ch 1), Cambridge

Elective M 311: Algebra 3

Syllabus:
Recapitulation of rings and modules : Noetherian and artinian rings and modules. Modules of finite length. Jordan-H\"{o}lder theorem. Krull-Schmidt theorem.
Tensor product --- definition, basic properties, right exactness, change of rings.
Semi-simple rings and modules. Wedderburn's theorems about structure of semi-simple and simple rings.
Nilradical and Jacobson radical. Nakayama's lemma. Jacobson radical of artinian ring is nilpotent. Ring semi-simple if and only if artinian and radical trivial. Artinian ring is noetherian.
Group representation (definition and generalities). Characters. Orthogonality relation. Burnside's two-prime theorem. Induced representation. Frobenius reciprocity.
Brauer's theorem on induced characters.

S. Lang - Algebra (Ch. 16, 17, 18), Springer
C. W. Curtis and I. Reiner - Representation Theory of finite Groups and Associative Algebras, Springer
P. M. Cohn - Further Algebra and Applications, Springer
T. Y. Lam - A First Course in Noncommutative Rings, Springer

Elective M 312: Commutative Algebra

Syllabus:
Zero divisors, Nilpotent elements, Nilradical and Jacobson radicals, Opertions on ideals , Extension and contraction, tensor product of modules, exactness properties of tensor products, Rings and modules of fractions, Primary Decomposition, Integral dependence

and valuations, chain conditions, Noetherian and Artinian rings, Discrete valuations and Dedekind domains, Completions, Dimension theory.

Suggested texts:
M. F. Atiyah & I. G. Macdonald-Introduction to Commutative Rings, Addison Wesley
D. Eisenbud-Commutative Algebra with a view towards Algebraic Geometry, Springer-
Miles Reid - Undergraduate Commutative Algebra, LMS 29, CUP
D. S. Dummitt and R. M. Foote – Abstract Algebra, Wiley, Ch. 15
N. S. Gopalakrishnan – Commutative Algebra, Oxonian Press

Elective M 313: Algebraic Geometry

Syllabus:

Affine algebraic sets: Affine spaces and algebraic sets, Noetherian rings, Hilbert basis theorem, affine algebraic sets as finite intersection of hypersurfaces; Ideal of a set of points, co-ordinate ring, morphism between algebraic sets, isomorphism. Integral extensions, Noether's normalization lemma, Hilbert's Nullstellensatz and applications: correspondence between radical ideals and algebraic sets, prime ideals and irreducible algebraic sets, maximal ideals and points, contrapositive equivalence between affine algebras with algebra homomorphisms and algebraic sets with morphisms, between affine domains and irreducible algebraic sets, decomposition of an algebraic set into irreducible components. Zariski topology on affine spaces, algebraic subsets of the plane.
Projective spaces: homogeneous co-ordinates, hyperplane at infinity, projective algebraic sets, homogeneous ideals and projective Nullstellensatz; Zariski topology on projective spaces. Twisted cubic in P_3(k).
Local properties of plane curves: multiple points and tangent lines, multiplicity and local rings, intersection numbers; projective plane curves: Linear systems of curves, intersections of projective curves: Bezout's theorem and applications; group structure on a cubic
Introduction to sheaves of affine varieties; examples of presheaves and sheaves, stalks, sheafification of a pre-sheaf, sections, structure sheaf, generic stalk and function fields, rational functions and local rings, Affine tangent spaces; Projective varieties and morphisms; Hausdorff axiom.
Prime spectrum of a ring: Zariski topology, structiure aheaf, affine schemes, morphism of affine schemes.
Elementary Dimension Theory, Fibres of a morphism, complete varieties, nonsingularity and regular local rings, Jacobian criterion, non-singular curves and DVR's.

Suggested texts:
W.Fulton - Algebraic curves, An introduction to algebraic geometry,
C. G. Gibson – Elementary Geometry of Algebraic Curves, CUP,
D. S. Dummitt and R. M. Foote – Abstract Algebra, Wiley, Ch. 15

Other texts:

J. Harris - Algebraic Geometry, A first course, Springer
M. Reid - Undergraduate algebraic geometry, LMS 12, CUP
K. Kendig – Elementary Algebraic Geometry, Springer
D. Mumford – The Red Book of Varieties and Schemes, Springer
I. R. Shafarevich – Basic Algebraic Geometry, Springer

Elective M 321: Algebraic and Differential Topology
Syllabus:
Topics from
Poincare Duality, Kunneth Product Formula, Universal Coefficient Theorem;
Differential topology: Transversality and Morse-Sard theorem,
Distributions and Integrability, Frobenius theorem;
Orientation, intersection number, Euler characteristic, Lefschetz fixed point theorem, Index of vector field, Poincare-Hopf index theorem;
Integration over chains; De Rham cohomology, De Rham's theorem, Stokes' theorem, Gauss map, Gauss-Bonnet theorem (if not already covered in Differential Geometry).
Characteristic Classes

Suggested texts:
Allen W. Hatcher - Algebraic Topology, Cambridge
J.W.Milnor and J. Stasheff- Characteristic Classes, HBA
V. Guillemin and A.Pollack - Differential Topology, Prentice Hall
M. Hirsch - Differential Topology, Springer
R. Bott and L.W.Tu - Differential Forms and Algebraic Topology, Springer

Elective M 322: Geometric Topology
Syllabus:
Topics from:
Knots and Links: Knot group, Seifert surfaces, Linking numbers, Alexander invariant, surgery on links;
Geometric structures – classification.

Hyperbolic Geometry: Models for hyperbolic space, Hyperbolic 2 manifolds;
Geometric group theory: Cayley graph of a group, Milnor-Svarc theorem, Quasi-isometries;

Hyperbolic groups in the sense of Gromov.

Suggested texts: J. Hempel - 3 manifolds, PUP
D. Rolfsen - Knots and Links, AMS
W. Jaco - Lectures on 3 manifold topology, AMS
R. Benedetti and C. Petronio - Lectures on Hyperbolic Geometry, Springer

W.P. Thurston - Geometry and Topology of 3 manifolds, Princeton Notes
S. M. Gersten (ed.) - Essays in Group Theory, Springer
E.Ghys and P. de la Harpe - Sur les groupes hyperbolique apers Mikhail Gromov, Birkhauser
E. Ghys, A. Haefliger, A. Verjovsky - Group theory from a geometrical viewpoint, World Scientific

Elective M 323: Lie groups and Lie Algebras

Syllabus:

Linear Lie groups, exponential map, Lie algebra of a Lie group, Lie subgroups and subalgebras, Lie transformation groups, coset spaces and homogeneous spaces, adjoint group, Invariant differential forms;
Lie algebras, nilpotent, solvable, semisimple Lie algebras, ideals, Killing form, Lie's and Engel's theorem, Universal enveloping algebra and Poincare-Birkhoff-Witt theorem;
Structure of semisimple Lie algebras, Cartan subalgebras, root space decomposition.

Suggested texts: J.E. Humphreys -Introduction to Lie algebras and representation theory, Springer
J.F. Adams - Lectures on Lie groups, Chicago
W. Knapp - Representation theory of semisimple groups, An overview based on examples;
W. Rossman - Lie groups: An Introduction through Linear groups. OUP

Elective M 324: Advanced Differential Geometry

Syllabus:

Jacobi Fields, conjugate points, Isometric immersions, Second fundamental form, Spaces of constant curvature, hyperbolic space, first and second variations of energy, Bonnet-Myers and Synge-Weinstein Theorems, Rauch comparison theorem, Morse Index theorem, Manifolds of negative curvature, Preissman's Theorem, Sphere theorem.

Suggested texts:
M.P. do Carmo - Riemannian Geometry (Ch. 5-13), Birkhauser
J.W. Milnor - Morse theory, PUP

Elective M 325: Complex Manifolds and Riemann Surfaces

Syllabus:

Cauchy's theorem in several complex variables, Weierstrass preparation theorem.
Definition of complex manifolds and Riemann surfaces, calculus on complex manifolds.
Sheaves and cohomology.
Divisors and Line bundles.
Normalization theorem.

Suggested texts:
Griffiths and Harris - Principles of Algebraic Geometry (Ch. 0, 1) -Wiley

Griffiths - Introduction to Algebraic Curves (Ch. 1-3) - AMS

Elective M 326: Complex Dynamics
Syllabus:
Revision of Universal coverings, Uniformization, Normal families, Montel's theorem.
Iterated Holomorphic maps: Fatou and Julia sets, dynamics on euclidean and hyperbolic surfaces, smooth Julia sets.
Fixed point theory: Attracting, repelling, indifferent fixed points. Parabolic fixed points and the Fatou flower, Cremer points.
Most periodic orbits repel, repelling cycles are dense in the Julia set.

Suggested texts:
J. Milnor – Dynamics in One Complex Variable, PUP.
A. Beardon - Iteration of Rational Maps - Springer
X. Buff and J. Hubbard - Complex Dynamics

Elective M 327: Advanced Algebraic Topology
Homotopy groups, Whitehead theorem, CW approximation, Freudenthal suspension theorem. Ref: Algebraic Topology: Hatcher

Serre spectral sequence, Calculations, Serre's theorem on homotopy groups of spheres. Ref: Spectral Sequences in Algebraic Topology: Hatcher

Vector bundles and characteristic classes. Ref: Characteristic classes: Milnor, Stasheff.

Generalised cohomology theory, K theory as an example, Bott periodicity, calculation of K theory, Atiyah Hirzebruch spectral sequence. Ref: Vector bundles and K theory: Hatcher.

Elective M 331: Logic and Set theory
Syllabus:
Naïve Set Theory: Relations and functions; Axiom of choice and Zorn's Lemma, Well-ordering principle, arithmetic of cardinal and ordinal numbers, transfinite induction.

Propositional calculus, Post's tautology theorem;
Predicate calculus, completeness theorems of predicate calculus; Godel numbers, recursive functions, Representability theorem
Godel's First Incompleteness Theorem.

Suggested texts:
S. M. Srivastava – A Course on Borel Sets, Springer;
H. Enderton – Introduction to Mathematical Logic, AP
J. Schoenfield – Introduction to Logic, AP

K. Kuratowski, H. Mostowski- Set Theory, van Nostrand
E. Mendelson – Introduction to Logic, AP
K. Kunen – Set Theory, Prentice Hall

Elective M 332: Programming and Data Structures

Syllabus:

Introduction: algorithms and programmes (notion of variables, actions, input/output); operational issues (editing, compiling, running, and debugging programmes).

C: variables, operators, expressions, statements, types (including some discussion on representation and size); control flow; arrays and pointers (notion of storage, memory locations, equivalence of pointers and arrays, pointer operations, multidimensional arrays, dynamic allocation/deallocation, strings); functions, macros, preprocessor directives, header files, multiple source files; structures and unions.

Data Structures: definition, lists (array and linked list implementations), stacks, queues, binary trees, tree traversal; elementary notions of time and space complexity, O-notation; sorting (radix or bucket, bubble or insertion, merge or quick); binary search, binary search trees; hashing.

Suggested texts:
B. Kernighan, D. Richie: The C Programming Language – PHI
J. Aho, H. Hopcroft, T. Ullman -Data Structures and Algorithms, Wiley
Other texts:
S. Gottfried - Programming in C, Schaum Series,
R.L. Kruse - Data Structures and Programme Design in C, PHI

Elective M 333: Graph theory and Combinatorics

Syllabus:

Inclusion-exclusion principle, pigeon-hole principle, Ramsey theorem for r-subsets classified into k classes.

Generating functions, recurrence relations, Polya's theory of counting, Ramsey Theory. Graphs, digraphs, degree, connectedness, paths, cycles, Eulerian and hamiltonian graphs, chromatic numbers, planarity, isomorphism,,subgraph, complement.

Regular graphs, strongly regular graphs, adjacency and incidence matrices, eigenvalues of graphs, Perron-Frobenius theorem, characterization of connectedness by spectrum, classification of graphs with largest eigenvalue at most 2 (Dynkin diagrams).

Suggested Texts:
C.L. Liu - Introduction to Combinatorial Mathematics
J. A. Bondy and U.S.R. Murty - Graph theory with applications

Other texts:
P. J. Cameron and J. H. van Lint - Designs, Graphs, Codes and their links, London Math. Soc., Students Text No. 22, Cambridge Univ. Press, 1991.
B. Bollobas - Random graphs, Acad. press.

F. Harary - Graph Theory
H.J. Ryser - Combinatorial Mathematics
M.J. Erickson - Introduction to Combinatorics

Elective M 334: Automata theory, Languages and Computability

Syllabus:

Automata theory: Finite automata, regular languages, regular expressions, equivalence of deterministic and non-deterministic finite automata, minimisation of finite automata, Kleene's theorem, pumping lemma, Myhill-Nerode theorem, Context-free grammar and languages, Chomsky normal form, pushdown automata, Context-sensitive languages, Chomsky hierarchy, closure properties.

Recursive, Primitive Recusive and partial recursive functions. Recursive and semirecursive (r.e.) sets, various equivalent models of Turing machines, Church-Turing thesis, Universal Turing machines and Halting Problem. Reducibility.

Complexity: Time complexity of deterministic and non-deterministic Turing machines, P and NP, Polynomial time reducibility, NP - completeness, Cook's theorem (statement only)

Suggested Texts:

J.E. Hopcroft and J.D.Ullman - Introduction to automata theory, languages and computation,
H.R.Lewis and C.H.Papadimitriou - Elements of the theory of computation

References:

S.M. Srivastava - A Course in Mathematical Logic, Springer
Martin Davis, R. Sigal and E. J. Weyuker - Computability, Complexity, and Languages: Fundamentals of Theoretical Computer Science

Elective M 341: Classical Mechanics 1

Lagrangian Mechanics, variational calculus, Lagrange's equations, Legendre transform, Liouville's theorem, holonomic principle, Noether's theorem, D'Alembert's principle.
Oscillations
Rigid bodies

Suggested Texts:

V.I. Arnold - Mathematical Methods of Classical Mechanics, Springer
R. Abraham and J. Marsden - Foundations of Mechanics, Addison-Wesley

Elective M 342: Classical Mechanics 2

Hamiltonian Mechanics, symplectic manifolds, symplectic atlas, Hamilton-Jacobi method, generating functions, Integrable systems

Suggested Texts:
V.I. Arnold - Mathematical Methods of Classical Mechanics, Springer
R. Abraham and J. Marsden - Foundations of Mechanics, Addison-Wesley

Elective M 343: Quantum Mechanics

Probability theory on the lattice of projections in a Hilbert space
Systems with a configuration under a group action
Multipliers on locally compact groups
The basic observables of a quantum mechanical system

Suggested Texts:
K.R. Parthasarathy - Mathematical Foundations of Quantum Mechanics. Hindustan Book Agency
S. J. Gustafson, I.M. Sigal - Mathematical Concepts of Quantum Mechanics, Springer

Project M400:
At the discretion of faculty, a student might be allowed to carry out a one semester project on an advanced topic. The project should be carried out in the 4th semester in lieu of one elective. The project is meant to inculcate in the student a taste for research.

Research Methodology M450: This course is a compulsory requirement for all PhD students. They will be tested in terms of their ability to use software for literature survey (MathSciNet) and prepare latex docments.

Special Topics M 500: Under special circumstances a special topic course might be offered by a faculty member. In such a case, the course content must be approved by the Board of Studies.

M 501: Topics in Topology
M 502: Topics in Geometry
M 503: Topics in Analysis
M 504: Topics in Dynamics
M 505: Topics in Algebra

Seminar M 601: Topology seminar
Seminar M 602: Analysis seminar
Seminar M 603: Algebra seminar
Research M 700
Note: In certain years, if there is a demand, the course on Number Theory (Course 13) may be shifted to the 3rd semester in lieu of an elective. In this case the slot in the 4th semester will become free for an elective.