SAMPLE MASTERS ENTRANCE EXAM MATHS RKMVERI

Note: all questions are MCQ/Short Answer type. Answers should be boxed and placed next to the question in the answersheet. The number of questions may vary.

(1) For a square nonsingular matrix A with real entries, pick the correct statement(s)
  (a) The eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A
  (b) 0 is not an eigenvalue of A
  (c) if A is symmetric the eigenvalues of A are real numbers.
  (d) if A is symmetric and α is an eigenvalue of A then $-\alpha$ is also an eigenvalue.

(2) Let π be a permutation in S_6 which is an 6-cycle. What is the cycle structure of π^2.

(3) For a linear operator T on a finite dimensional $\mathbb{R}$ vector space V, pick the correct statement(s)
  (a) If T has an eigenvector x with eigenvalue λ and P is a polynomial with real coefficients then x is an eigenvector of $P(T)$ with eigenvalue $P(\lambda)$.
  (b) If $V = \mathbb{R}^n$ and T is rotation by $\frac{\pi}{2}$ radians then each non-zero vector is an eigenvector for T^2
  (c) If T^2 has a non-negative eigenvalue λ^2 at least one of λ or $-\lambda$ is an eigenvalue of T.
  (d) If x and y are two eigenvectors of T with distinct eigenvalues and $ax + by$ is also an eigenvector of T then $a = 0$ or $b = 0$.

(4) If $f(x, y) = \frac{\sin(x^2+y^2)}{x^2+y^2}$ whenever $(x, y) \neq (0, 0)$, how must $f(0, 0)$ be defined so as to make it continuous at the origin.

(5) What is the maximal number of pieces into which a solid cube can be divided by 4 straight cuts?

(6) Pick the correct statement(s)
  (a) If an abelian group has elements of orders m and n respectively then it has an element whose order is the least common multiple (lcm) of m and n.
  (b) If an abelian group has subgroups of orders m and n then it has a subgroup of order equalling the lcm of m and n.

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(c) If G is a finite group whose order is not divisible by 3 such that $(ab)^3 = a^3b^3$ for all $a, b \in G$. Then G cannot be abelian.
(d) If all subgroups of a group G are normal then G is abelian.

(7) Find the number of ring homomorphism from $\mathbb{Z}[X, Y]$ to $\mathbb{F}_2[X]/(X^3+X^2+X+1)$.

(8) The curve $f(x) = \dfrac{x^3}{1+x^2}$ has asymptotes
(a) y-axis
b) y=x
(c) x-axis
(d) None of these

(8) Let G be a group in which, for some integer $n > 1$, $(ab)^n = a^nb^n$ for all $a, b \in G$ Pick the correct choices
(a) $G^{(n)} := \{x^n \mid x \in G\}$ is a normal subgroup of G.
(b) $G^{(n-1)} := \{x^n \mid x \in G\}$ is a normal subgroup of G.
(c) $a^{n-1}b^n = b^na^{n-1}$ for all $a, b \in G$
(d) None of the above

(9) The integral $\int_0^x \frac{\sin t}{t+1}$ is
(a) nonnegative for all $x \geq 0$
(b) is negative for x an odd multiple of π
(c) is nonnegative only for $x \leq \pi$
(d) none of the above

(10) Let the function f be defined for all real x as $f(x) = x^2$ if x is rational and $f(x) = 0$ if x is irrational. Calculate $f'(0)$ provided that it exists.