Numerical index of a Banach space

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Abstract

Numerical index of a Banach space is a number relating the norm and the numerical range of a bounded linear operator. Given a Banach space X, the *numerical radius* of a bounded linear operator T on X is given by

$$v(T) := \sup\{|x^*(Tx)| : x^* \in X^*, x \in X, \|x^*\| = \|x\| = x^*(x) = 1\}$$

and the *numerical index* of X is given by $n(X) := \inf\{v(T) : T \in \mathcal{L}(X), \|T\| = 1\}$.

The problem of computing the numerical index of the L_p-spaces has been latent since the beginning of the theory. It is known that $n(L_p(\mu)) = \inf\{n(l_p^m) : m \in \mathbb{N}\}$ for every measure μ such that $dim(L_p(\mu)) = \infty$. In this talk I will start from basics of numerical index and will be showing that

$$n(l_p^2) = \sup_{t\in[0,1]} \frac{|t^{p-1} - t|}{1 + t^p} \text{ for } p \in [1.4547, 3.19925].$$

This result is an extenstion of the result by Javier Merí and Alicia Quero. The computation of two dimensional case is the first step in the computation of $n(l_p)$, but it is reasonable to expect that the sequence $\{n(l_p^m)\}$ is always constant as it happens in the cases $p = 1, 2, \infty$. So computation of $n(l_p^2)$ is most important step and the above result is the best result known so far.