Ramakrishna Mission Vivekananda Educational and Research Institute

PO Belur Math, Howrah, West Bengal 711 202

School of Mathematical Sciences

Department of Mathematics

M.Sc. Mathematics Entrance Test 2024 - Written Examination

Date: 5 May 2024

Time : 2 p.m to 5 p.m

SET-A

Total Marks: 90 Duration: 3 hours

Instructions for the candidate:

- The questions are of **multiple choice** type.
- Each question has **4** options.
- Every question has **one** correct option.
- Each question carries **5 marks**.
- **2 mark** will be deducted for every wrong answer.
- **No rough work is required for the purpose of evaluation, and this question paper and OMR sheet should not be used for rough work.** Separate sheets will be provided for doing rough work.
- At the end of the examination, submit question paper, rough work and the OMR sheet.

Instructions for filling the OMR sheet:

- The OMR sheet should not be folded or crushed.
- Use **only blue/ black ball point** pen to fill the circles. Do not use marker or white fluid to hide the mark.
- Use of pencil is strictly prohibited.
- **Marking more than one option will be treated as a wrong answer.** You will be provided only one OMR sheet. So mark carefully.

Correct Methods Wrong Methods

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1. Which of the following metric spaces X has the property that for every set-theoretic function

$$f : X \to \{0, 1\},$$

either the set $f^{-1}(0)$ or the set $f^{-1}(1)$ has two points at a distance 1.

A. $X = \mathbb{R}$, the set of real numbers,

B. $X = S^1 := \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$, the unit circle,

C. $X = \mathbb{R}^2$, the Euclidean plane,

D. none of the above.

2. Let $A \in \mathcal{M}_{m\times n}(\mathbb{R})$ and $B \in \mathcal{M}_{n\times m}(\mathbb{R})$ be real matrices such that $AB = I_m$, the identity matrix. Then which of the following statements is correct?

A. The rows of A are linearly independent, but the columns of B can be linearly dependent.

B. The rows of A can be linearly dependent, but the columns of B are linearly independent.

C. The rows of A can be linearly dependent and the columns of B can be linearly dependent.

D. The rows of A are linearly independent and the columns of B are linearly independent.

3. Let $S := \{\sin(n^2 - 2n + 4) \mid n \in \mathbb{Z}\} \cap \mathbb{Z}$. Then

A. S is an infinite set.

B. $S = \{0, 1, -1\}$.

C. $S = \{0\}$.

D. none of the above is true.

4. Which one among the following pairs of groups are isomorphic?

A. $(\mathbb{R}, +)$ and $(\mathbb{C}, +)$

B. $(\mathbb{R}, +)$ and $(\mathbb{C}^*, \cdot)$

C. $(\mathbb{Q}, +)$ and $(\mathbb{Q}^+, \cdot)$

D. $(\mathbb{Z}, +)$ and $(\mathbb{Z}^2, +)$

5. Let $H := \{(2n, 4n) \in \mathbb{Z}^2 \mid n \in \mathbb{Z}\}$, and G be the quotient group of $\mathbb{Z}^2$, defined as $G := \mathbb{Z}^2/H$. Then

A. every element of G has a finite order.

B. G is not abelian.

C. every element of G other than the identity element has an infinite order.

D. none of the above is true.

6. The symmetric group S_4 on 4 letters does not have a subgroup which isomorphic to

A. S_3

B. $\mathbb{Z}_2 \times \mathbb{Z}_3$

C. $\mathbb{Z}_3$

D. D_4

7. Let $A \in \mathcal{M}_n(\mathbb{R})$ be a nilpotent matrix, i.e., there exists a positive integer r such that $A^r = 0$. Now let us consider the following two statements.

   (i) If I denotes the $n \times n$ identity matrix, then $I + A$ is invertible.

   (ii) If $B \in \mathcal{M}_n(\mathbb{R})$ is another nilpotent matrix, then $A + B$ is also nilpotent.

   A. both (i) and (ii) are true.

   B. (i) is true, but (ii) is false.

   C. both (i) and (ii) are false.

   D. (i) is false, but (ii) is true.

8. Let R be a commutative ring with identity. For every $a \in R$, we define a map $h_a : R \to R$, given by $h_a(r) := ar$ for all $r \in R$. Now let us consider the following two statements.

   (i) For each $a \in R$, if h_a is injective, then it must be surjective.

   (ii) For each $a \in R$, if h_a is surjective, then it must be injective.

   A. (i) is false, but (ii) is true.

   B. both (i) and (ii) are true.

   C. both (i) and (ii) are false.

   D. (i) is true, but (ii) is false.

9. Which of the following statements is true?

   A. $\sum_{n=1}^{\infty} \sin \frac{1}{n}$ is convergent.

   B. $\sum_{n=1}^{\infty} \tan \frac{1}{n}$ is convergent.

   C. $\sum_{n=1}^{\infty} \cos \frac{1}{n^2}$ is convergent.

   D. None of the above.

10. How many of the following statements are correct?

    (i) If $f : \mathbb{R} \to \mathbb{R}$ is the function defined as

$$f(x) := \begin{cases} \sin x \cdot \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{otherwise,} \end{cases}$$

    then f is differentiable at $x = 0$.

    (ii) If $g : \mathbb{R} \to \mathbb{R}$ is the function defined as

$$g(x) := \begin{cases} \sin x^2 \cdot \sin \frac{1}{x^2} & \text{if } x \neq 0 \\ 0 & \text{otherwise,} \end{cases}$$

    then g is differentiable at $x = 0$.

    (iii) If $h : \mathbb{R} \to \mathbb{R}$ is the function defined as

$$h(x) := \begin{cases} \cos x \cdot \cos \frac{1}{x} & \text{if } x \neq 0 \\ 1 & \text{otherwise,} \end{cases}$$

    then h is continuous at $x = 0$.

    A. 2

    B. 3

    C. 1

    D. 0

11. The last digit of $1^1 + 2^2 + 3^3 + 4^4 + 5^5 + 6^6 + 7^7 + 8^8 + 9^9 + 10^{10}$ is

    A. 3

    B. 1

    C. 7

    D. 9

12. You climb a staircase, where you are allowed to take either 1 or 2 stairs at a time. In how many ways can you climb a staircase with 10 stairs?

    A. 88

    B. 87

    C. 89

    D. 90

13. Let S be the subgroup of $(\mathbb{Q}, +)$, defined as

$$S := \left\{ \frac{n}{2^j} \mid n \in \mathbb{Z},\ j \in \mathbb{N} \right\} \subseteq \mathbb{Q}.$$

    Then which one of the following is true?

    A. S is isomorphic to $\mathbb{Q}$.

    B. S is a finitely generated subgroup of $\mathbb{Q}$.

    C. S has finite index in $\mathbb{Q}$.

    D. None of the above.

14. Let the sequence $(x_n)_{n\in\mathbb{N}}$ be recursively defined as $x_1 := 1$ and $x_{n+1} := 1 + x_n^{-1}$ for all $n \in \mathbb{N}$. This sequence is

    A. monotone, but not convergent.

    B. not monotone, but convergent.

    C. both monotone and convergent.

    D. neither monotone nor convergent.

15. Let the sequence of functions $f_n : \mathbb{R} \to \mathbb{R}$ be recursively defined as $f_1(x) := \cos x$ and $f_{n+1}(x) := \cos(f_n(x))$ for all $n \geq 2$. Then which of the following statements is false?

    A. For every natural number n, $f_n(x)$ has a fixed point, i.e., there exists a point $x_n \in \mathbb{R}$ such that $f_n(x_n) = x_n$.

    B. The sequence of functions $(f_n(x))_{n\in\mathbb{N}}$ is point-wise convergent.

    C. For every natural number n, $f_n(x)$ has infinitely many zeros.

    D. For every natural number n, $f_n(x)$ is a periodic function.

16. If $f : \mathbb{R} \to \mathbb{R}$ is a continuous function, then which of the following statements must always be correct?

    A. If $U \subseteq \mathbb{R}$ is an open set, then $f(U)$ is also open.

    B. If the composition $f^2 := f \circ f$ has a fixed point, then f has a fixed point too.

    C. If for every integer $n \in \mathbb{Z}$, there exists a polynomial function $p_n \in \mathbb{R}[X]$ such that $f|_{[n,n+1]} = p_n|_{[n,n+1]}$, then f itself is a polynomial function, i.e., $f \in \mathbb{R}[X]$.

    D. If $C \subseteq \mathbb{R}$ is a closed set, then $f(C)$ is also closed.

17. Let $(\mathrm{GL}_2(\mathbb{R}), \cdot)$ denote the group of all 2×2 invertible matrices over $\mathbb{R}$, the field of real numbers. Now let us consider the following two statements.

(i) $\mathrm{GL}_2(\mathbb{R})$ contains infinitely many elements of finite order.

(ii) If $A \in \mathrm{GL}_2(\mathbb{R})$ is an element of finite order, then $A^2 = I$, where I is the 2×2 identity matrix.

Then

A. both (i) and (ii) are false.

B. (i) is false, but (ii) is true.

C. (i) is true, but (ii) is false.

D. both (i) and (ii) are true.

18. Let f, g be two functions from $\mathbb{R}$ to $\mathbb{R}$ such that the composition $g \circ f$ is continuous. Then which of the following statements must be true?

A. g is continuous.

B. f is continuous.

C. Either f or g is continuous.

D. None of the above.

Question	Key
1	C
2	D
3	D
4	A
5	D
6	B
7	B
8	A
9	D
10	C
11	C
12	C
13	D
14	B
15	C
16	B
17	C
18	D