Abstract

Automorphisms and Representations of Quantum Groups
by
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This thesis investigates the automorphisms of the quantum affine space $\mathcal{O}_{\mathfrak{q}} = \mathbb{K}_{q_{ij}}[X_1, \ldots, X_n]$. Necessary and sufficient conditions on the multiparameters q_{ij} are obtained so that the only $\mathbb{K}$-automorphisms are the trivial ones arising from the action of the torus $(\mathbb{K}^*)^n$. Additionally, the automorphism group is computed when the localization, the quantum torus $\widehat{\mathcal{O}}_{\mathfrak{q}} = \mathbb{K}_{q_{ij}}[X_1^{\pm 1}, \ldots, X_n^{\pm}]$ has Krull dimension one.

Quantum tori of Krull dimension one hold particular significance. It is known that quantum tori with Krull dimension one arise when the subgroup Λ generated by the multiparameters q_{ij} has maximal rank $\binom{n}{2}$. Explicit examples of quantum tori with Krull dimension one and small Λ-rank are constructed and relate them to the invariant $s_n(K)$ which is defined by the simultaneous vanishing of families of skew-forms on subspaces of a K-vector space. The study of $s_n(K)$ naturally leads to the consideration of n-subspaces of $\mathrm{Alt}_n(K)$. In this direction decompositions of $\mathrm{Alt}_n(K)$ are obtained as a direct sum of n-subspaces thus refining and enhancing the results of R. Gow and R. Quinlan.

The inequality of Bernstein plays a key role in the theory of D-modules. In the quantum case Bernstein-type inequalities have been established for quantum Weyl algebras.The work is extended in this direction by establishing Bernstein-type inequalities for the algebras $K_{n,\Gamma}^{P,Q}(\mathbb{K})$, introduced by K. L. Horton which include the graded quantum Weyl algebra, the quantum symplectic space, and the quantum Euclidean space. This approach leads to a simpler proof of this inequality for the quantum Weyl algebra.

Furthermore, in this thesis, the two-parameter quantum Heisenberg enveloping algebra, which serves as a model for certain quantum generalized Heisenberg algebras, has been studied at roots of unity. In this context, the quantum Heisenberg enveloping algebra becomes a polynomial identity algebra and the dimension of simple modules is bounded by its PI degree. A detailed investigation is carried out to determine the PI degree, describe the center, and provide a complete classification of simple modules up to isomorphism.

Keywords: Simple Modules, Ambiskew Polynomial Rings, Gelfand-Kirillov Dimension, Quantum Torus, Quantum Affine Spaces, Automorphisms, Alternating Forms, Skew-Symmetric Forms, Constant Rank Spaces, Cyclic Extensions, Matrices, Rank, Dimension.