ABSTRACT

The goal of this thesis is twofold:

1. Diophantine approximation problems with prime denominator in quadratic number fields under GRH.

    - The problem in imaginary quadratic number fields of both class number 1 and greater than 1.

    - The problem in real quadratic number fields of both class number 1 and greater than 1.

    This is the main substance of the thesis.

2. Diophantine approximation problem with squarefree denominator in imaginary quadratic number field of class number 1.

In the 1930s, I. M. Vinogradov [53] demonstrated the existence of infinitely many primes p satisfying $\left|\alpha - \frac{b}{p}\right| < p^{-1-1/5+\varepsilon}$, for a suitable $b \in \mathbb{Z}$. Since then, numerous researchers have refined and extended this result. In 2009, Matomäki proved that if $\alpha \in \mathbb{R}$ is irrational, then there are infinitely many primes p such that $|\alpha - a/p| \leq p^{-4/3+\varepsilon}$ for a suitable integer a. This is considered to be the limit of current tools and techniques. In this thesis, we extend this result to all quadratic number fields under the condition that the Grand Riemann Hypothesis holds for their Hecke L-functions. First, we prove this for the easier