PHD THESIS ABSTRACT

We prove asymptotic formulae for small weighted solutions of quadratic congruences of the form $\lambda_1 x_1^2 + \cdots + \lambda_n x_n^2 \equiv \lambda_{n+1} \bmod p^m$, where p is a fixed odd prime, $\lambda_1, ..., \lambda_{n+1}$ are integer coefficients such that $(\lambda_1 \cdots \lambda_n, p) = 1$ and $m \to \infty$. If $n \geq 6$, $p \geq 5$ and the coefficients are fixed and satisfy $\lambda_1, ..., \lambda_n > 0$ and $(\lambda_{n+1}, p) = 1$ (inhomogeneous case), we obtain an asymptotic formula which is valid for integral solutions $(x_1, ..., x_n)$ in cubes of side length at least $p^{(1/2+\varepsilon)m}$, centered at the origin. If $n \geq 4$ and $\lambda_{n+1} = 0$ (homogeneous case), we prove a result of the same strength for coefficients λ_i that are allowed to vary with m. These results extend the previous results of S. Baier, A. Haldar and N. Bag.

In the fifth chapter, we consider a hybrid problem of calculating the variance of primes in intersections of arithmetic progressions and short intervals in the function field setting. Keating and Rudnick derived asymptotic formulas for the variances of primes in arithmetic progressions and short intervals. They used an involution to translate short intervals into arithmetic progressions. We follow their approach but apply this involution, in addition, to the arithmetic progressions. This creates dual arithmetic progressions in the case when the modulus Q is a polynomial in $\mathbb{F}_q[T]$ such that $Q(0) \neq 0$. The latter is a restriction which we keep throughout the chapter.