A Study of Lie-Yamaguti Algebra and its Bundle

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Abstract

In the present thesis, we study Lie-Yamaguti algebras and Lie-Yamaguti algebra bundles. Lie-Yamaguti algebras are a class of non-associative algebras equipped with a bilinear operation and a trilinear operation satisfying some intriguing relations involving the operations. These algebras arose naturally in the study of reductive homogeneous spaces.

We develop algebraic deformation theory of Lie-Yamaguti algebras following the methodology of Gerstenhaber's deformation theory of associative algebras.

We use Lie-Yamaguti algebra cohomology to study the inducibility problem for automorphisms of abelian extensions of Lie-Yamaguti algebras, measuring obstruction to lift a given automorphism to its extension as a cohomology class. We derive Wells exact sequence in the context of extensions of Lie-Yamaguti algebras. As an application, we describe automorphism groups of certain semi-direct product Lie-Yamaguti algebras and analyze the inducibility problem for nilpotent Lie-Yamaguti algebras of index 2. We construct infinite families of such examples and provide an algorithm to determine all inducible pairs of automorphisms in these cases.

Next, we construct a universal commutative algebra associated with a finite-dimensional Lie-Yamaguti algebra, extending the idea of Manin-Tambara's universal constructions for associative algebras. Furthermore, we show that the resulting universal algebra admits a natural Hopf algebra structure. This leads to the definition of a universal coacting Hopf algebra for a given Lie-Yamaguti algebra. We also provide a representation-theoretic perspective of the construction. As applications, we describe the automorphism group and provide a classification of all abelian group gradings of a finite-dimensional Lie-Yamaguti algebra.

Finally, we introduce the notion of Lie-Yamaguti algebra bundle, define its cohomology groups with coefficients in a representation. Such bundles appeared implicitly in the work of M. Kikkawa, to address certain geometric properties of locally reductive spaces. This motivates us to introduce Lie-Yamaguti algebra bundles. We also study abelian extensions of Lie-Yamaguti algebra bundles and investigate their relationship with suitable cohomology group.