MSc Programme in mathematics at RAMAKRISHNA MISSION VIVEKANANDA Educational and Research Institute for AY 2025 - 26 onwards

Semester 1:

- Course No 1. M 200 Introduction to Analysis (4 credits)
- Course No 2. M 201 Algebra 1 (5 credits)
- Course No 3. M 209 Elementary Number Theory (5 credits)
- Course No 4. M 204 Linear Algebra (4 credits)
- Course No 5. M 216: Foundations of Real Number System (2 credits)

Semester 2:

- Course No 5. M 203 Complex Analysis (5 credits)
- Course No 6. M 210 Algebra 2 (5 credits)
- Course No 7. M 202 Topology 1 (5 credits)
- Course No 8. M 205: Real Analysis 1 (5 credits)

Semester 3:

- Course No 9. M 211 Functional Analysis (5 credits)
- Course No 10. M 207 Linear Algebra 2 (5 credits)
- Course No 11. Elective 1 (4 credits)
- Course No 12. Real Analysis 2 (5 credits)

Semester 4:

- Course No 13. Elective 2 (4 credits)
- Course No 14. Elective 3 (4 credits)
- Course no. 15. Elective 4 (4 credits)
- Course No 16. M 400 Project (8 credits)
- Course No 17. M 220- Problem Solving (1 credit)

Note: Among the three electives in the course, one must be an Interdisciplinary/Applied Math elective.

Coursewise Syllabi

Notes:

1. *In the various courses described below the instructor should stick to at least 70% of the syllabus. The remaining 30% can be shaped and tailored according to the instructor to suit his/her individual philosophy of teaching as well the the demands of the students or any other special situation that may arise.*
2. *Wherever there is an overlap/interconnection between concurrent courses running in the same semester the respective instructors will try to maximize the synergy of such an interconnection.*
3. *An elective course from the Interdisciplinary/Applied Math stream must be opted and passed by the masters student. The list of possible electives is appended to the following syllabi for courses.*
4. *In the third semester the student can choose a course of applied nature. In case this course is offered from the math dept (e.g, Coding Theory, Discrete Mathematics, Cryptography etc.) it will normally be a 5 credit course.*

Semester 1

M 200: introduction to Analysis
Syllabus: basic concepts in analysis sequences/series, infinite products, uniform convergence, metric space concepts, Riemann integral.
(*if time permits*) Topology of $\mathbb{R}^n$, uniform convergence in $\mathbb{R}^n$.
Suggested Text:

- Principles of Mathematical Analysis by W. Rudin
- Elementary Classical Analysis by Jerrold E. Marsden

M 201: Algebra 1
Syllabus:
Group Theory
Group action on a Set, Stabilisers and Orbits, Burnside–Frobenius lemma, transitive action, Cayley's theorem, Class Equation, Automorphisms, Sylow's Theorems, Direct Products, Symmetric and Alternating groups,
If time permits, Semidirect Products, Groups of Platonic solids. Polya enumeration
Combinatorial Group Theory
Free groups, presentation of groups by generators
If time permits, Todd Coxeter Algorithm, Free products.
RingTheory
Examples, Ring Homomorphisms, Ideals, Quotient Rings with emphasis on Z/nZ and Modular Arithmetic, Isomorphism Theorems, Properties of Ideals,

Prime and Maximal Ideals, Rings of Fractions, Chinese remainder theorem; Euclidean domains, PID, UFD, Factorization in Z[i].

Polynomials and Polynomial Rings : Definition and basic properties, universal mapping property, division algorithm; R[x] is a UFD if R is a UFD; Irreducibility Criteria; Noetherian conditon.

If time permits,

Hilbert basis theorem and applications; Definitions and examples of Algebraic Sets and Coordinate rings. Polynomial Maps.

Suggested Text:

- E. Vinberg - A course in Algebra, Graduate Studies in Mathematics.Other texts:

Other texts:

- M. Artin - Algebra - PHI
- D.S. Dummit and R.M. Foote - Abstract Algebra J. Wiley
- I. N. Herstein- Topics in Algebra; J. Wiley
- N. Jacobson - Basic Algebra Ch. 1,2. Van Nostrand

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M 204: Linear Algebra 1

Syllabus:

Quotients of vector spaces, Geometric significance of cosets and quotient spaces, Dimension of Quotients of Vector Space; Linear Transformations: Kernel, Image, Isomorphisms, rank and nullity, linear functionals, annihilators, Dual and Double dual, Transpose of a Linear Transformation. Eigenvectors of a linear transformation. Inner Product Spaces, Hermitian, Unitary and Normal Transformations, Spectral Theorems, Bilinear and Quadratic Forms.

Note: 1) It is desirable to hold problem sessions, so that the students gaina firm conceptual grasp as well as the capacity to solve problems. The initial sessions are to review basic topics like elementary matrices and elementary operations, invertible and elementary matrices, matrix concepts like similarity and rank and their relation to linear transformations. 2) For the topic Eigenspaces, it suffices to cover basic concepts necessary for Spectral theory. Minimal and characteristic polynomials, canonical forms will be done in Linear Algebra 2. 3) Geometric significance of concepts to be emphasized wherever necessary, e.g. geometric interpretation of specific linear operators, orthogonal matrices, determinant as volume, etc.

Suggested Text:

- K. Hoffman and R. Kunze – Linear Algebra, PHI, Ch. 3 (with revision of 1), 6.1-6.2, 8, 9,10.

Other texts:

- D.S. Dummit and R.M. Foote - Abstract Algebra, John Wiley - Ch. 11
- M. Artin - Algebra, PHI – Ch. 4, 7.
- N. S. Gopalakrishnan – University Algebra, Wiley Eastern, Ch. 3.5, 5.1-5.7, 5.11-5.13.
- I. N. Herstein – Topics in Algebra, John Wiley, Ch. 4.1-4.4, 6.1-6.3, 6.8-6.11.

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M 209: Elementary Number Theory
Syllabus
Note: Upto to 50% of this course can be modeled based on the choice of the instructor/demands of students so as to lay emphasis on a particular aspect of the subject. For example, the course could be geared towards analytic number theory or combinatorial number theory etc. The following syllabusfor a sample.

Divisibility Theory in Integers; Congruences: Euler-Fermat, Wilson's Theorem, Chinese Remainder Theorem, quadratic reciprocity law; Sum of two squares, Arithmetical functions - average order of some functions; Distribution of Primes: Chebyshev's bound, elementary proof of prime number theorem.

Additive number theory: Schnirelman's Theorem, Cauchy-Davenport Theorem, EGZ Theorem, van der Waerden's Theorem, Diophantine approximation: Liouville's Theorem.
Suggested books:

- Elementary Number theory by David M. Burton
- An Introduction to the Theory of Numbers by G.H. Hardy, E.M. Wright.
- Additive Number Theory: Inverse Problems and the Geometry of Sumsets by Nathanson, Melvyn B

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M 216: Foundations of Real Number System
Syllabus
Axiom of Choice and Zorn's Lemma (discussion only). Countable and Uncountable Sets–(2 hrs).
Cardinal Arithmetic using Zorn's Lemma–(4 hrs).
Metric spaces with examples. Open and closed sets, Continuous function, Sequences, Cauchy Sequences and their convergence.–(5 hrs).
Complete Metric Spaces, Cantor Intersection Theorem, Baire Category Theorem, Cantor Ternary sets.–(7 hrs)
Compact and Sequential Compact spaces, Continuous functions on metric spaces,

Lebesgue Lemma, Totally bounded metric spaces, Equivalent definitions of Compact metric spaces. Tychonoff Theorem for Countable Product of metric spaces. Characterization of Compact sets in Euclidean spaces.–(10 hrs).

Suggested books:

- S. M Srivastava, A Course on Borel Sets, GTM 180, Springer, 1997.

Semester 2

M 202: Topology 1

Syllabus

Topological Spaces and Continuous Functions: Basis, Order and Product Topology, Closed Sets and Limit Points; Metric Topology, Completion of Metric Spaces, Baire Category Theorem; Product Topology; Connectedness and Compactness: Connectedness and local connectedness, compactness and local compactness; Separation and countability axioms, T1, T2, T3, T4, Urysohn Lemma, Tietze extension theorem; Tychonoff theorem. Quotient topology and identification spaces; Topological manifolds as examples of quotient topology - torus, Klein's bottle, projective spaces

; Examples of topological groups; Homotopy of paths, Fundamental Group; Covering spaces and group actions on spaces, computation of fundamental group of the circle. Fundamental groups of surfaces.

Suggested Texts:

- J.R. Munkres - Topology Ch. 2 (sec 12-20), 3 (sec 2-29), 4(sec. 30-35), 5(sec. 37)
- sec 22 of Ch 2 for quotient topology, section on topological groups, Ch. 9, sec 51-55, 58-60 for Fundamental Group); PHI
- G.F. Simmons - Topology and Modern Analysis (ch. 2 sec. 9-13 for metric spaces); TMH
- M. A. Armstrong - Basic Topology; Springer

Other texts:

- I. Singer and J. Thorpe - Notes on Elementary Topology and Differential Geometry. Springer
- P. J. Higgins – An Introduction to Topological Groups – LMS lecture notes CUP
- J.W. Milnor - Topology from a differentiable viewpoint (for notion of manifolds and classification of 1 and 2-manifolds) PUP
- S.M. Srivastava - A Course on Borel Sets, Springer

M 205: Real Analysis 1
Syllabus:
Periodic functions and Fourier Series, Trignometric functions, Measure and Lebesgue measure
Suggested Texts:

- Fourier Analysis - An Introduction (Princeton Lectures in Analysis, Volume 1) by Elias M. Stein and Rami Shakarchi

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M 210: Algebra 2 -Fields and Galois Theory
Syllabus:
Finite groups, simple groups, solvable groups, simplicity of A_n.

Field Theory: Algebraic Extensions, Finite and algebraic extensions, Normal extensions, algebraic closure, separable and inseparable extensions, primitive element; Galois theory: Galois extensions and Galois group, Galois' Theorem, fundamental theorem; Explicit examples and concrete applications of Galois theory, Resolvents; Roots of unity, cyclotomic polynomials and extensions, solvability by radicals, Abel's theorem, finite fields;

If time permits:
Introduction to Transcendental Extensions: Finite transcendence degree. Integral Extensions; Applications to Ruler and Compass constructions. Lemniscate division.

Note: The topic on finite groups should be done just before the topic Solvability by radicals. Section 4.6 of Jacobson's Basic Algebra 1, or Section 5.7 of Herstein should be used.
Suggested text:

- D.S. Dummit and R.M. Foote -Abstract Algebra, Wiley - Ch. 13, 14, 15.1 – 15.3
- N.S.Gopalakrishnan - University Algebra, Wiley Eastern – Ch. 4
- TIFR pamphlet on Galois theory.
- Other texts:
- S. Lang – Algebra, Addison Wesley, (Ch. 5, 6.1 – 6.7, 7.1, 8.1, 9.1);
- I. N. Herstein – Topics in Algebra, John Wiley, Ch. 5.
- N. Jacobson - Basic Algebra 1, HBA, Ch. 4
- G. Rotman – Galois Theory, Springer.
- **D. Cox - Galois Theory , Wiley.**

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M 203: Complex Analysis
Syllabus:
Analytic Functions: Power series, Trigonometric functions, Cauchy - Riemann equations, analytic functions as mappings; Complex integration: Cauchy's theorem and integral formula, power series representation, zeros of an analytic function, Meromorphic functions and residue calculus, Index of a closed curve, Morera's theorem, Liouville's theorem, open mapping theorem; Singularities: Classification, Rouche's theorem, argument principle; Maximum modulus principle, Schwarz lemma, analytic continuation; Compactness and convergence in the space of analytic functions: Space of continuous functions, space of analytic functions, normal families, space of meromorphic functions, Riemann mapping theorem

Suggested text:

- J.B. Conway - Functions of One Complex Variable: Narosa

Other texts:

- T.W. Gamelin - Complex Analysis, Springer
- L. V. Ahlfors - Complex Analysis, TMH
- W. Rudin - Real and Complex Analysis TMH
- S. Ponnasamy- Complex Analysis TMH
- D.E. Sarason - Complex Function Theory HBA

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Semester 3

M 206: Real Analysis 2
Syllabus:
Multivariable Calculus:
Functions of several variables: Differentiation, Contraction Principle, Inverse Function Theorem, Implicit Function Theorem; Rank Theorem; Jacobians, Differentiation of Integrals;
Ordinary Differential Equations
Ordinary Differential Equations: Cauchy-Peano Existence Theorem, Uniqueness, Picard-Lindelof Theorem; Continuation of solutions, systems of differential equations;

Suggested Texts:

- Calculus II by Tom Aopstol

- Elementary Classical Analysis by Jerrold E. Marsden
- E. A. Coddington and N. Levinson - Theory of Ordinary Differential Equations (1.1-1.6), PHI

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ME 1 Elective 1 (Interdisciplinary/Applied Math)
(see below the list of third semester electives)

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M 207: Linear Algebra 2
Syllabus:
Concept of a module. Modules over Commutative Rings and submodules. Examples: Vector Spaces; Abelian Groups, Commutative Rings; Ideals and Quotients, Invariant subspaces of a K-linear transformation of a vector space V as a K[X] submodule of V; Module Homomorphisms, Kernel and Image, Hom(M,N), Generation of modules, Direct sum and Free Modules. Exact sequences. Noetherian modules, Annihilator and torsion submodules; Characterization of rings in terms of modules; Finitely generated modules over PID, submodule of a free module is free;

Structure theorems – Invariant factor form and elementary divisor form; Primary decomposition theorems, (proof of uniqueness may be omitted) Application to abelian groups.

Introduction to Canonical Forms: Statements and Applications; Outline of Proofs to be given. Details of Proofs may be excluded from the examination syllabus. Minimal and characteristic polynomials; triangularisation over algebraically closed field; Cayley-Hamilton Theorem, Nilpotent transformations. Rational and Jordan canonical forms.

Suggested Texts:

- D.S. Dummit and R.M. Foote -Abstract Algebra J. Wiley
- N.S.Gopalakrishnan - University Algebra Oxonian Press
- K. Hoffman and R. Kunze – Linear Algebra, PHI (Ch 3.7- 3.10, Ch 8-10)
- M. Artin - Algebra, PHI
- E. Vinberg - A course in algebra, GSM.

Other texts:

- I. N. Herstein – Topics in Algebra, John Wiley, Ch. 6.4-6.7.
- N. Jacobson - Basic Algebra 1, HBA, Ch. 3

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M 211: Functional Analysis

Syllabus (Functional Analysis)

Revision of measure theory

Normed Linear Spaces and Banach Spaces: Bounded linear operators, Duals, Hahn- Banach theorem; Uniform boundedness principle; Open mapping and Closed Graph theorems, some applications; Dual spaces: Computing duals of $L^p(1 \leq p < \infty)$ and $C[0,1]$; reflexive spaces; Weak and weak* topologies, Banach Alaoglu theorem. Hilbert Spaces - Orthogonal sets, Projection theorem, Riesz representation theorem, Adjoint operator; Self-adjoint, normal and unitary operators, Projections. Spectrum and spectral radius; Spectral theorem for compact operators. If time permits, Spectral theorem for self-adjoint, normal and unitary operators;

Suggested Text:

- G. F. Simmons - Topology and Modern Analysis (Ch. 9, 10, 11, 12), TMH
- J. B. Conway - A First Course in Functional Analysis, Springer

Other texts:

- W. Rudin – Real and Complex analysis TMH

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Semester 4

M 220: Problem Solving

The course shall comprise of problem sessions across the topics of Linear Algebra, Abstract algebra, Real amd Complex Analysis, Basic Topology and Differential Equations.

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ME 2: Elective 2

See the list of 4th semester electives.

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ME 3: Elective 3

See the list of 4th semester electives.

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ME 4: Elective 4

See the list of 4th semester electives.

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Project M400
Note This course is intended to be a one-semester project work to undertaken with a faculty member. The student can read a topic of interest with the approval/guidance of his mentor/supervisor (who will normally be a faculty member of the department). While this course is meant to promote independent study on the part of the student, the student will be required to give a 25 - 35 minutes board/projector presentation at the end of the semester open to all the members of the mathematics department. There will be a five 5 -7 minutes question/answer period followed by the presentation. The students will be also required to submit typed notes of their presentation. The notes should be typed in Latex.

A major part (70%) of the assessment and evaluation for this course should be based on the the typed notes as well as presentation talk given by the student. The mentor of the student can assign the grade for this course.

3rd semester electives

M 213 Discrete Mathematics

Syllabus

Basic concepts of set theory, cardinal numbers, mathematical induction, pigeonhole principle, permuations and combinations, inclusion-exclusion principle, recurrence relations, generating functions, Polya's theorem, graphs, trees, matching: Hall's marriage theorem, Ramsey theory, planar graph, Partially ordered set: Dilworth's theorem and extremal set theory

suggested texts:

- M. Aigner, Discrete Mathematics, AMS
- van Lint and Wilson, A course in Combinatorics, Cambridge Univ. Press
- Martin J. Erickson, Introduction to Combinatorics, Wiley

M214: Cryptography

Syllabus

Public-key cryptography: RSA, ElGamal; Protocols: Diffie-Hellman, Fiat-Shamir; Elliptic curve cryptosystem

Suggested texts

- Neal Koblitz, A course in Number Theory and Cryptography, Springer
- Delfs and Knebl: Introduction to Cryptography, Springer

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CS221 - Design and Analysis of Algorithms

Syllabus

- 1. Different order notations like O, Θ, Ω, θ, ω and compare two different functions using order notation.
- 2. Methods to calculate and state running time of algorithms using order notations.
- 3. Divide and Conquer paradigm of algorithm design through its application in devising algorithms for merge sort, counting inversions, finding closest pair of points in a plane, fast integer multiplication, fast Fourier transform etc.
- 4. Dynamic Programming and use of memoization through several examples like longest increasing subsequence, edit distance, knapsack, matrix chain multiplication, independent sets in trees etc.

- 5. Greedy methods of algorithm design through various examples like minimum spanning trees, huffman codes, horn clauses etc.
- 6. Breadth First Search (BFS) and Depth First Search (DFS) in graphs.
- 7. Application of BFS and DFS like topological sorting of a directed acyclic graph, finding all strongly connected components of a directed Graph, finding articulation points,bridges and biconnected component of a graph, finding Eulerian tour in a Eulerian graph.
- 8. Kruskal and Prim's algorithm for minimum spanning trees and union find data structure.
- 9. Algorithms for single source shortest paths in a directed graph like Bellman-Ford algorithm, Dijkstra's algorithm.
- 10. Algorithms for all pair shortest paths like the matrix multiplication based procedure, Floyd-Warshall algorithm, Johnson's algorithm for sparse graphs.
- 11. Complexity class NP, NP-Completeness, NP-Hardness, reducibility.

Suggested texts

- Jon Kleinberg, Eva Tardos; Algorithm Design, Pearson education, 2008
- A. Aho, J. Hopcroft and J. Ullman; The Design and Analysis of Computer Algorithms, A.W.L, International Student Edition, Singapore, 1998
- S. Dasgupta, C.H. Papadimitriou, and U.V. Vazirani; Algorithms, Tata McGraw-Hill c2008
- S. Baase: Computer Algorithms: Introduction to Design and Analysis, 2nd ed., Addison-Wesley, California, 1988.
- T. H. Cormen, C.E. Leiserson and R.L.Rivest: Introduction to Algorithms, Prentice Hall of India, New Delhi, 1998.
- E. Horowitz and S. Sahni: Fundamental of Computer Algorithms, Galgotia Pub./Pitman, New Delhi/London, 1987/1978.
- K. Mehlhorn: Data Structures and Algorithms, Vol. 1 and Vol. 2, Springer-Verlag, Berlin, 1984.
- A. Borodin and 1. Munro: The Computational Complexity of Algebraic and Numeric Problems, American Elsevier, New York, 1975.
- D.E. Knuth: The Art of Computer Programming, Vol. 1, Vol. 2 and Vol. 3. Vol. 1, 2nd ed., Narosa/Addison-Wesley, New Delhi/London, 1973; Vol. 2: 2nd ed., Addison-Wesley, London, 1981; Vol. 3: Addison-Wesley, London, 1973.

- S. Winograd: The Arithmetic Complexity of Computation, SIAM, New York, 1980.

CS 244 : Introduction to Optimization Techniques
Syllabus
Mathematical Preliminaries:
Theory of Sets and Functions, Vector spaces, Matrices and Determinants, Convex sets and convex cones, Convex and concave functions, Generalized concavity

Linear Programming :
The (Conventional) Linear Programming Model The Simplex Method: Tableau And Computation Special Simplex Method And Implementations Duality And Sensitivity Analysis

Integer Programming Formulating Integer Programing Problems Solving Integer Programs (Branch-and-Bound Enumeration, Implicit Enumeration, Cutting Plane Methods)

Nonlinear Programming: Theory Constrained Optimization Problem (equality and inequality constraints) Necessary and Suffiecent conditions Constraint Qualification Lanrangian Duality and Saddle Point Optimality Criteria

Nonlinear Programming: Algorithms The concept of Algorithm Algorithms for Uconstrained Optimization Constraint Qualification Algorithms for Constrained Optimization (Penalty Function, Barrier Function, Feasible Direction)
Special Topics (if time permits) Semi-definite and Semi-infinte Programs Quadratic Programming Linear Fractional programming Separable Programming *suggested texts*

- Linear programming and Network Flows, Wiley-Blackwell; 4th Edition, 2010
- M. S. Bazaraa, John J. Jarvis and Hanif D. Sheral, ISBN-13: 978-0470462720
- • Nonlinear Programming: Theory and Algorithms, Wiley-Blackwell; 3rd Edition (2006)
- M. S. Bazaraa, Hanif D. Sherali, C. M. Shetty, ISBN-13: 978-047148600

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AM200: Nonlinear Dynamics and Asymptotic Analysis
Syllabus
Linearization- 1D systems Bifurcations - Saddle node bifurcation, Transcritical bifurcation, Pitchfork bifurcation
2D systems- Bifurcation of fixed points – Saddle node Transcritical Pitchfork

Bifurcation of periodic orbits – Coalescence of orbits Saddle node in invariant circle Homoclinic bifurcation

Chaos- Strange Attractor, 1D Map, Period Doubling

Asymptotic analysis and Perturbation Theory Regular perturbation, Singular perturbation, Method of dominant balance, Big O, small o, Differential Equations, Boundary layer theory, WKB approximation
Suggested texts

- Nonlinear Dynamics and Chaos, Steven H. Strogatz, CRC Press.
- Advanced Mathematical methods for scientists and engineers, C. Bender and S. Orszag, Springer.

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AM 201 Numerical Algorithms
Syllabus Numerics and Error analysis Floating Point representation Machine Epsilon Absolute error and Relative error, Backward error Forward error

Solutions of Nonlinear equation Fixed point iteration Bisection method Newton Raphson method Secant method

Numerical Optimization Method of Golden section search Newton's optimization method

Solutions for linear algebraic equations Forward Gauss elimination Back Substitution LU Decomposition

Interpolation Lagrange interpolation Newton interpolation

Numerical Integration Finite Difference Trapezoidal rule Simpsons rule Gaussian quadrature Numerical solutions to Ordinary Differential Equations

Suggested text: Numerical Recipes: The Art of Scientific Computing, Cambridge University Press

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CS312: Approximation and Online Algorithms
syllabus
Approximation Algorithm: Performance Measure, Greedy Algorithm, Unweighted Vertex Cover Problem Minumum-degree Spanning Tree, Minimum Weight Spanning Tree, The Traveling-Salesman Problem,The k-Center Problem, Multi-way Cut and K-Cut Problems, Scheduling Jobs with Deadlines on a Single Machine, Scheduling Jobs on Identical Parallel Machines, The Set Cover Problem, An

Application of Set Cover to Art Gallery problems, Shortest Super-string Problem Rounding Data and Dynamic Programming, The Knapsack Problem, The Bin-Packing Problem, The Primal-Dual Method, Weighted Vertex Cover Problem.

Online Algorithms: Competitive Analysis, The Paging Problem, Amortized Analysis, List Update Problem, Scheduling Jobs on Identical Parallel Machines, Graph Coloring, Machine Learning, K-Server Problem, Target Searching in an Unbounded Region and Target Searching in Streets

References:

- M. R. Garey and D. S. Johnson, Computers and Intractability: A guide to the theory of NP-completeness, W. H. Freeman, 1979.
- R. Motwani, Lecture Notes on Approximation Algorithms, Volume 1, No. STAN-CS-92-1435, Stanford University, 1992.
- D. P. Williamson and D. B. Shmoys, The Design of Approximation Algorithms, Cambridge University Press, 2011.
- Vijay Vazirani, Approximation algorithms, Springer-Verlag, 2001.
- S. Albers, Competitive Online Algorithms, Lecture notes, Max Plank Institute, Saarbrucken, 1996.
- S. K. Ghosh and R. Klein, Online algorithms for searching and exploration in the plane, Computer Science Review, vol. 4, pp. 189-201, 201

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CS312: Computing for Data Science

Syllabus

Definition of computing, Binary representation of numbers intergers, floating point, text.

Unconventional / application specific file formats, like media. Bitmap representation for monochromatic image and generalizing the representation for RGB. File metadata, Speed of CPU, Memory, Secondary storage, DMA. Using and understanding the

Basics of Linux. Hardisk organization into Cylinder, Track, and Sectors for storing data.

Learning programming using Python. arrays([], [][]), conditional structures (if), and iterative structures (while, for), defining functions, using library functions. Programming assignment:

Dictionary data structure in python, File access in python, Sorting and Searching algorithms, appreciating complexity of algorithms. Programming using numerical methods. Basics of Turing machine as a model of computing, analysing the performance of a program, time complexity, space complexity, difference between efficiency and performance, Analyse the first sorting algorithm.

Basic notations of complexity like Big Oh, omega etc, and their mathematical definitions, given a programme to compute the complexity measures.

Programming in SQL (Structured query language) to query relational databases. Representation of graphs, basic algorithms like minimum spanning tree, matching etc. Monte-Carlo simulation Object oriented programming using Java

Suggested texts

- Algorithms in Data Science, First edition Brian Steele, John Chandler, & Swarna Reddy
- How to proram in Python Louden & Louden
- How to proram in Java Louden & Louden

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Sem 4 electives

M212: Topology 2
Syllabus

Singular homology and Eilenberg-Steenrod Axioms: Relative Homology, excision and exactness. Mayer-Vietoris sequence, homotopy invariance; Cellular homology as an example of a homology theory: Computation of homology for cell-complexes like S^n, CP^n, closed 2-manifolds. Singular cohomology, cup and cap products, Cohomology ring; Poincare duality for closed manifolds.
Suggested texts:

- A. Hatcher - Algebraic Topology (Ch. 2,3)
- J.R. Munkres - Elements of Algebraic Topology, Addison Wesley

Elective M 300: Coding Theory:
Syllabus:

Golay Code, Cyclic Codes, Codes over Z_4, Goppa Codes, Algebraic Geometry Codes.

Books:

- van Lint, Introduction to Coding Theory, Springer
- Huffman and Vera Pless, Fundamentals of error - correcting codes, Cambridge
- Ling, and Xing, Coding Theory: A first course, Cambridge

Elective M 301: Advanced Complex Analysis
Syllabus:
1. Further applications of Cauchy's theorem. (a) Morera's theorem. (b) Sequences of holomorphic functions. (c) Holomorphic functions defined in terms of integrals. (d) Schwartz refelection principle. 2. Meromorphic functions and the logarithm. (a) Zeros and poles. (b) the residue formula-examples. (c) Singularities and meromorphic functions. (d) the argument principle and applications. (e) Homotopies and simply connected domains. (f) The complex logarithm. (g) Fourier series and harmonic functions. 3. Entire functions-(a) Jensen's formula. (b) functions of finite order. (c) Infinite products-Generalities. Example. The product formula for the sine function. (d) Hadamard's factorisation theorem. 4. The Gamma and zeta functions.
Suggested texts:

- E.M. Stein and R. Shakrachi. Complex Analysis. Vol. 2. Princeton lectures in Analysis

- L.V. Ahlfors - Complex Analysis TMH

Other texts

- J. B. Conway - Functions of one Complex Variable (Ch. 7-12) Narosa
- H.M. Farkas and I. Kra - Riemann surfaces, Springer

Elective M 302: Harmonic Analysis
Syllabus:
Topological groups, quotients and products, open subgroups, Haar Measure on Locally Compact Groups; Properties of Haar measure, Invariant measures on homogeneous spaces. Representation of compact Lie groups, Schur's Lemma, Weyl character formula, Pete- Weyl theory, Representations of SU(2,C). If time permits, Induced representation and Frobenius reciprocity theorem, Principal series representations of Sl(2,R)

Suggested texts:

- G.B. Folland: Introduction to Abstract Harmonic Analysis, CRC
- Press S.C. Bagchi, S. Madan, A. Sitaram and U.B. Tewari - A first course on representation theory and linear Lie groups, University Press.
- A. Deitmar - A first course in harmonic analysis, Springer
- E. Stein and R. Shakarchi – Fourier Analysis, PUP

Elective M 303: Probability Theory
Syllabus:
Independence, Kolmogorov zero-one law, Kolmogorov three-series theorem, Strong law of large numbers, Levy-Cramer continuity, Central limit theorem, Infinite products of probability measures, Discrete-time discrete state Markov chains.
Suggested texts:

- P. Billingsley - Probability and measure
- Y.S. Chow and H. Teicher - Probability theory, Independence, interchangeability, martingales.
- J . Nevue - Mathematical foundations of the calculus of probability

Elective M 304: Distribution Theory
Syllabus: C^∞ functions on R^n, smooth partition of unity on R^n; Test function space on an open subset Ω of R^n; Space of distributions on Ω, functions and measures as distributions; Examples of distributions on Ω that do not extend to distributions on R^n; Elementary operations on the space of distributions: Derivatives of distributions, multiplication by a function, convolution by a test function; Sequences of distributions: convergence and approximation by test functions.

Schwartz space, Isomorphism of Schwartz space with itself under Fourier transform; Fourier inversion and Fourier-Plancherel Theorem; Tempered distributions, Fourier transforms of tempered distributions; Distributions of compact support; Convolution of a tempered distribution with a function of Schwartz class; Fourier transform of derivatives and convolutions (with a Schwartz class function) of tempered distributions;

Application of distributions to solving PDE's: Weak solutions, some easy examples, statement of elliptic regularity; solution of Laplace equation on the half-plane and the Heat equation in R^3, using Fourier transforms.

Suggested Texts:

- W. Rudin – Functional Analysis, TMH, Ch. 6, 7.1 – 7.19
- R. Strichartz – A guide to distribution theory and Fourier transforms, CRC Press, 1994, Ch. 5
- G.B. Folland – Fourier Analysis and its applications, Wadsworth and Brooks
- S. Kesavan: Functional analysis and applications, John Wiley, 1989

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Elective M 305: Operator Algebras
Syllabus
Multiplicative functionals and maximal ideal space, Gelfand transform, Gelfand Naimark theorem, Rational functional Calculus

C* Algebras, Positive Cones of C* Algebras States and GNS Construction (If time permits, some of the following topics in C* algebras may be touched upon Approximate Identitites, Extreme points on the unit ball, Pure States and regular maximal ideals Ideals, Quotients and Representations)

Banach space of operators on a Hilbert space B(H) Locally convex Topologies on B(H) Polar decomposition and orthogonal decomposition von Neumann Double Commutant Theorem If time permits, Kaplansky's density theorem

Suggested Texts:

- R. V. Kadison and J. R. Ringrose – Fundamentals of the Theory of Operator Algebras, AMS.

Other texts:

- Introduction to Operator Algebras – Li Bing Ren, World Scientific
- W. Arveson – An Invitation to C* Algebras (Ch 1), Springer
- V. S. Sunder - An Invitation to von Neumann Algebras (Ch 1), Cambridge

M307: Analytic Number Theory
Syllabus:

1. **Arithmetic functions and Dirichlet series:** the ring of arithmetic functions, Dirichlet series, important arithmetic functions, average estimates.

2. **Characters:** group characters, Dirichlet characters, detection of residue classes, Gauss sums.

3. **Prime number distribution:** infinitude of primes, Chebyshev's bounds, Riemann zeta function, Perron's formula, prime number theorem, Dirichlet L-functions, primes in arithmetic progressions.

4. **Circle method:** general setup, ternary Goldbach problem, partitions.

5. **Sieve methods:** Selberg's sieve, large sieve, estimates for twin primes, estimates for twins of almost-primes.

6. **Extra topics:** modular forms, exponential sums.

Main Text Book:

- T.M. Apostol, *Introduction to Analytic Number Theory*, Springer 1998.

Supplimentary Books:

1. H.L. Montgomery, R.C. Vaughan, *Multiplicative Number Theory: I. Classical Theory*, Cambridge University Press 2007.
2. H. Davenport, *Multiplicative Number Theory*, Springer 2000.
3. H. Iwaniec, I. Kowalski, *Analytic Number Theory*, AMS 2004.
4. R.C. Vaughan, *The Hardy-Littlewood method*, Cambridge University Press 1997.
5. A.C. Cojucaru, R. Murty, *Introduction to Sieve Methods and their Applications*, Cambridge University Press 2006.
6. S.W. Graham, G. Kolesnik, *Van Der Corput's Method of Exponential Sums*, Cambridge University Press 1991.
7. N. Koblitz, *Introduction to Elliptic Curves and Modular Forms*, Springer 1993.

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M306: Algebraic Number Theory

Unique factorization, Primitive roots and Group structure of $U(Z/nZ)$, Quadratic reciprocity, Quadratic Gauss sums, Finite Fields, Gauss and Jacobi sums, Cubic and biquadratic reciprocity, equations over finite fields, Unique factorization in Algebraic number fields, ramification and degree, unit theorem, quadratic and cyclotomic fields. *suggested texts*

- Ireland and Rosen, A classical introduction to modern number theory, Springer
- TIFR Pamphlet, Algebraic Number Theoy
- P. Samuel, Algebraic Theory of Numbers
- Lang, Algebraic Number Theory

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M 308: Differential Geometry

Syllabus:

Manifolds: Smooth functions, vector fields, Jacobian, integral curves, submanifolds; Connections and curvature for surfaces in R^3, Gauss map.

The classical theory of surfaces in R^3 to be stressed and done in detail as the first set of examples where the notions of connection and curvature come up. The general theory below to be described in the context of Riemannian manifolds only.

Riemannian manifolds and submanifolds: Length and distance, Riemannian connection and curvature, curves, submanifolds, hypersurfaces.

Operators on forms and integration: Exterior derivative, contraction, Lie derivative, general covariant derivative, integration of forms and Stokes' theorem; Surfaces in R^3, Gauss-Bonnet formula and Index theorem.

Suggested texts:

- N.J. Hicks - Notes on Differential Geometry; Ch. 1, 2, 3, 7, 8.1, 8.2, AP (Ch 5,6 of the above reference deal with the theory of connections and curvature in great detail and can be used as a reference for these topics, rather than a text. For this topic it is advisable to use Ch 2 and 4 of the text below as the basic text.)
- M. P. do Carmo - Riemannian Geometry (Ch. 1,2,3,4) Birkhauser

Other texts:

- S. Kumaresan, A Course on Differential Geometry and Lie Groups, HBA
- N.J. Hicks - Notes on Differential Geometry; Ch. 5, 6 AP
- B. O'Neill - Elementary Differential Geometry; Springer
- Klingenberg - Elementary Differential Geometry; AP
- M. P. do Carmo - Differential geometry of curves and surfaces Birkhauser
- M. Spivak - Calculus on Manifolds Publish or Perish
- Singer and Thorpe – Notes on Elementary Topology and Differential Geometry Springer

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Elective M 311: Algebra 3

Syllabus:

Recapitulation of rings and modules : Noetherian and artinian rings and modules. Modules of finite length. Jordan-Hölder theorem. Krull-Schmidt theorem. Tensor product - definition, basic properties, right exactness, change of rings. Semi-simple rings and modules. Wedderburn's theorems about structure of semi-simple and simple rings. Linear representations, Semisimplicity, Characters, Algebras, Matrix algebras, Quaternion algerbras, Group algebras, Introduction to Central Simple algebra and the Brauer group, Finite Dimensional Algebras, Tensor product, Symmetric and Grassmann algebra, Fundamentals of Lie groups and Lie algebras, Categories and Functor. (*if time permits*) Basics of Homological Algebra.

Suggested text:

- Jacobson N., Basic Algebra I and II, Dover Books.
- Vinberg E., A course in Algebra, American Math. Soc.
- Rowen L., Graduate Algebra: Noncommutative View, Graduate Studies in Mathematics.

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Elective M 312 : Commutative Algebra
Syllabus:
Zero divisors, Nilpotent elements, Nilradical and Jacobson radicals, Opertions on ideals , Extension and contraction, tensor product of modules, exactness properties of tensor products, Rings and modules of fractions, Primary Decomposition, Integral dependence and valuations, chain conditions, Noetherian and Artinian rings, Discrete valuations and Dedekind domains, Completions, Dimension theory.

Suggested texts:

- M. F. Atiyah & I. G. Macdonald-Introduction to Commutative Rings, Addison Wesley
- D. Eisenbud-Commutative Algebra with a view towards Algebraic Geometry, Springer- Miles Reid - Undergraduate Commutative Algebra, LMS 29, CUP
- D. S. Dummitt and R. M. Foote – Abstract Algebra, Wiley, Ch. 15
- N. S. Gopalakrishnan – Commutative Algebra, Oxonian Press

Elective M 313: Algebraic Geometry
Syllabus:
Affine algebraic sets: Affine spaces and algebraic sets, Noetherian rings, Hilbert basis theorem, affine algebraic sets as finite intersection of hypersurfaces; Ideal of a set of points, co-ordinate ring, morphism between algebraic sets, isomorphism. Integral extensions, Noether's normalization lemma, Hilbert's Nullstellensatz and applications: correspondence between radical ideals and algebraic sets, prime ideals and irreducible algebraic sets, maximal ideals and points, contrapositive equivalence between affine algebras with algebra homomorphisms and algebraic sets with morphisms, between affine domains and irreducible algebraic sets, decomposition of an algebraic set into irreducible components. Zariski topology on affine spaces, algebraic subsets of the plane. Projective spaces: homogeneous co-ordinates, hyperplane at infinity, projective algebraic sets, homogeneous ideals and projective Nullstellensatz; Zariski topology on projective spaces. Twisted cubic in $P_3(k)$.

Local properties of plane curves: multiple points and tangent lines, multiplicity and local rings, intersection numbers; projective plane curves: Linear systems of curves, intersections of projective curves: Bezout's theorem and applications; group structure on a cubic Introduction to sheaves of affine varieties; examples of presheaves and sheaves, stalks, sheafification of a pre-sheaf, sections, structure sheaf, generic stalk and function fields, rational functions and local rings, Affine tangent spaces; Projective varieties and morphisms; Hausdorff axiom. Prime spectrum of a ring: Zariski topology, structiure aheaf, affine schemes, morphism of affine schemes. Elementary Dimension Theory, Fibres of a morphism, complete varieties, nonsingularity and regular local rings, Jacobian criterion, non-singular curves and DVR's.

Suggested texts:

- W.Fulton - Algebraic curves, An introduction to algebraic geometry,
- C. G. Gibson – Elementary Geometry of Algebraic Curves, CUP,
- D. S. Dummitt and R. M. Foote – Abstract Algebra, Wiley, Ch. 15

Other texts:

- J. Harris - Algebraic Geometry, A first course, Springer
- M. Reid - Undergraduate algebraic geometry, LMS 12, CUP
- K. Kendig – Elementary Algebraic Geometry, Springer
- D. Mumford – The Red Book of Varieties and Schemes, Springer
- I. R. Shafarevich – Basic Algebraic Geometry, Springer

Elective M 322: Geometric Topology
Syllabus:
Topics from: Knots and Links: Knot group, Seifert surfaces, Linking numbers, Alexander invariant, surgery on links; Geometric structures – classification.

Hyperbolic Geometry: Models for hyperbolic space, Hyperbolic 2 manifolds; Geometric group theory: Cayley graph of a group, MilnorSvarc theorem, Quasi-isometries;

Hyperbolic groups in the sense of Gromov.
Suggested texts:

- J. Hempel - 3 manifolds, PUP
- D. Rolfsen - Knots and Links, AMS
- W. Jaco - Lectures on 3 manifold topology, AMS
- R. Benedetti and C. Petronio - Lectures on Hyperbolic Geometry, Springer
- W.P. Thurston - Geometry and Topology of 3 manifolds, Princeton Notes
- S. M. Gersten (ed.) - Essays in Group Theory, Springer

- E.Ghys and P. de la Harpe - Sur les groupes hyperbolique apers Mikhail Gromov, Birkhauser
- E. Ghys, A. Haefliger, A. Verjovsky - Group theory from a geometrical viewpoint, World Scientific

Elective M 323: Lie groups and Lie Algebras
Syllabus:
Linear Lie groups, exponential map, Lie algebra of a Lie group, Lie subgroups and subalgebras, Lie transformation groups, coset spaces and homogeneous spaces, adjoint group, Invariant differential forms; Lie algebras, nilpotent, solvable, semisimple Lie algebras, ideals, Killing form, Lie's and Engel's theorem, Universal enveloping algebra and Poincare-Birkhoff-Witt theorem; Structure of semisimple Lie algebras, Cartan subalgebras, root space decomposition.

Suggested texts:

- J.E. Humphreys - Introduction to Lie algebras and representation theory, Springer
- J.F. Adams - Lectures on Lie groups, Chicago
- W. Knapp - Representation theory of semisimple groups, An overview based on examples;
- W. Rossman - Lie groups: An Introduction through Linear groups. OUP

Elective M 324: Advanced Differential Geometry
Syllabus:
Jacobi Fields, conjugate points, Isometric immersions, Second fundamental form, Spaces of constant curvature, hyperbolic space, first and second variations of energy, Bonne-Myers and Synge-Weinstein Theorems, Rauch comparison theorem, Morse Index theorem, Manifolds of negative curvature, Preissman's Theorem, Sphere theorem.

Suggested texts:

- M.P. do Carmo - Riemannian Geometry (Ch. 5-13), Birkhauser
- J.W. Milnor - Morse theory, PUP

Elective M 325: Complex Manifolds and Riemann Surfaces
Syllabus:
Cauchy's theorem in several complex variables, Weierstrass preparation theorem. Definition of complex manifolds and Riemann surfaces, calculus on complex manifolds. Sheaves and cohomology. Divisors and Line bundles. Normalization theorem.

Suggested texts:

- Griffiths and Harris - Principles of Algebraic Geometry (Ch. 0, 1) – Wiley

- Griffiths - Introduction to Algebraic Curves (Ch. 1-3) - AMS

Elective M 326: Complex Dynamics
Syllabus:
Revision of Universal coverings, Uniformization, Normal families, Montel's theorem. Iterated Holomorphic maps: Fatou and Julia sets, dynamics on euclidean and hyperbolic surfaces, smooth Julia sets. Fixed point theory: Attracting, repelling, indifferent fixed points. Parabolic fixed points and the Fatou flower, Cremer points. Most periodic orbits repel, repelling cycles are dense in the Julia set.

Suggested texts:

- J. Milnor – Dynamics in One Complex Variable, PUP.
- A. Beardon - Iteration of Rational Maps - Springer
- X. Buff and J. Hubbard - Complex Dynamics

Elective M 327: Advanced Algebraic Topology
Syllabus
Homotopy groups, Whitehead theorem, CW approximation, Freudenthal suspension theorem. Ref: Algebraic Topology: Hatcher

Serre spectral sequence, Calculations, Serre's theorem on homotopy groups of spheres. Ref: Spectral Sequences in Algebraic Topology: Hatcher

Vector bundles and characteristic classes. Ref: Characteristic classes: Milnor, Stasheff.

Generalised cohomology theory, K theory as an example, Bott periodicity, calculation of K theory, Atiyah Hirzebruch spectral sequence.

Ref:

- Vector bundles and K theory, Hatcher.

Elective M 331: Logic and Set theory
Syllabus:
Naïve Set Theory: Relations and functions; Axiom of choice and Zorn's Lemma, Well- ordering principle, arithmetic of cardinal and ordinal numbers, transfinite induction.

Propositional calculus, Post's tautology theorem; Predicate calculus, completeness theorems of predicate calculus; Godel numbers, recursive functions, Representability theorem Godel's First Incompleteness Theorem.

Suggested texts:

- S. M. Srivastava – A Course on Borel Sets, Springer;
- H. Enderton – Introduction to Mathematical Logic, AP
- J. Schoenfield – Introduction to Logic, APK. Kuratowski, H. Mostowski-Set Theory, van Nostrand

- E. Mendelson – Introduction to Logic, AP
- K. Kunen – Set Theory, Prentice Hall

Elective M 332: Programming and Data Structures
Syllabus: Introduction: algorithms and programmes (notion of variables, actions, input/output); operational issues (editing, compiling, running, and debugging programmes). C: variables, operators, expressions, statements, types (including some discussion on representation and size); control flow; arrays and pointers (notion of storage, memory locations, equivalence of pointers and arrays, pointer operations, multidimensional arrays, dynamic allocation/deallocation, strings); functions, macros, preprocessor directives, header files, multiple source files; structures and unions. Data Structures: definition, lists (array and linked list implementations), stacks, queues, binary trees, tree traversal; elementary notions of time and space complexity, O-notation; sorting (radix or bucket, bubble or insertion, merge or quick); binary search, binary search trees; hashing.

Suggested texts:

- B. Kernighan, D. Richie: The C Programming Language – PHI
- J. Aho, H. Hopcroft, T. Ullman -Data Structures and Algorithms, Wiley
- Other texts:
- S. Gottfried - Programming in C, Schaum Series,
- R.L. Kruse - Data Structures and Programme Design in C, PHI

Elective M 334: Automata theory, Languages and Computability
Syllabus:
Automata theory: Finite automata, regular languages, regular expressions, equivalence of deterministic and non-deterministic finite automata, minimisation of finite automata, Kleene's theorem, pumping lemma, Myhill-Nerode theorem, Context-free grammar and languages, Chomsky normal form, pushdown automata, Context-sensitive languages, Chomsky hierarchy, closure properties.

Recursive, Primitive Recusive and partial recursive functions. Recursive and semirecursive (r.e.) sets, various equivalent models of Turing machines, Church-Turing thesis, Universal Turing machines and Halting Problem. Reducibility.

Complexity: Time complexity of deterministic and non-deterministic Turing machines, P and NP, Polynomial time reducibility, NP - completeness, Cook's theorem (statement only)

Suggested Texts:

- J.E. Hopcroft and J.D.Ullman - Introduction to automata theory, languages and
- computation,
- H.R.Lewis and C.H.Papadimitriou - Elements of the theory of computation

References:

- S.M. Srivastava- A Course in Mathematical Logic, Springer
- Martin Davis, R. Sigal and E. J. Weyuker - Computability, Complexity, and Languages:
- Fundamentals of Theoretical Computer Science

Elective M 341: Classical Mechanics 1
Syllabus
Lagrangian Mechanics, variational calculus, Lagrange's equations, Legendre transform, Liouville's theorem, holonomic principle, Noether's theorem, D'Alembert's principle. Oscillations Rigid bodies
Suggested Texts:

- V.I. Arnold - Mathematical Methods of Classical Mechanics, Springer
- R. Abraham and J. Marsden - Foundations of Mechanics, Addison-Wesley

Elective M 342: Classical Mechanics 2
Syllabus
Hamiltonian Mechanics, symplectic manifolds, symplectic atlas, Hamilton-Jacobi method, generating functions, Integrable systems
Suggested Texts:

- V.I. Arnold - Mathematical Methods of Classical Mechanics, Springer
- R. Abraham and J. Marsden - Foundations of Mechanics, Addison-Wesley

Elective M 343: Quantum Mechanics
Syllabus
Probability theory on the lattice of projections in a Hilbert space Systems with a configuration under a group action Multipliers on locally compact groups The basic observables of

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a quantum mechanical system
Suggested Texts:

- K.R. Parthasarathy - Mathematical Foundations of Quantum Mechanics. Hindustan Book Agency
- S. J. Gustafson, I.M. Sigal - Mathematical Concepts of Quantum Mechanics, Springer

Elective M 344: Classical representation theory of finite groups - I
Syllabus
Linear representations of finite groups, low-dimensional examples, one dimensional representations, subrepresentations and quotient representations, direct

sum, contragredient and tensor product of representations, group algebra, modules and representations,minimal polynomial, Maschke's Theorem, Semisimplicity, Clifford's theorem extension of scalars, endomorphism algebra, biendomorphism algebra, Schur's lemma, minimal left ideals, idempotents, structure of Hom, Wedderburn decomposition, primitive idempotents, characters and class functions, principal inner product on group algebra, orthogonality relations, computation of character table for S_n for small n and Dihedral groups
if time permits
representation theory of symmetric groups, Young Tableux, Hook-length formula

Elective M 347: Measure Theory
Syllabus
An introduction to abstract measure theory and the Lebesgue integral. Lebesgue integral, The main convergence theorems, Fatou39;s Lemma, Monotone Convergence Theorem, Dominated Convergence theorem, Egoroff39;s theorem, Vitali39;s convergence theorem, Convergence in measure, convergence in L p. Construction Lebesgue measure in R n . L p - spaces, Radon-Nikodym Theorem, Lebesgue Differentiation Theorem, Fubini Theorem. *Suggested Texts:*

- H. Royden and P. Fitzpatrik, Real Analysis,
- W. Rudin, Real and Complex Analysis.

Elective M 348: Fourier Analysis
Syllabus
Part 1: Fourier series : Dirichlet's problem, Fourier series, Fejer's theorem, point-wise convergence of Fourier series under different conditions on the function, L^2 theory, Plancherel theorem, Applications.

Part 2. Fourier transform : Fourier transform on $\mathbb{R}^n$, its properties, Riemann-Lebesgue lemma, Schwartz class functions, L^2 theory, Plancherel theorem, Fourier transform of L^p functions, $1 \leq p \leq 2$, Hausdorff-Young's inequality, Applications. *Suggested Texts:*

- Principles to mathematical analysis, W. Rudin.
- Mathematical analysis, T. M. Apostol.

Elective M 351: Nonlinear Dynamics and Chaos:
Syllabus
Introduction, Terminology and applicability: (a) General idea of dynamical system, order of dynamical system, continuous and discrete, rheonomous and autonomous systems. (b) One-dimensional systems: Flows on the line. Fixed points and stability, graphical analysis, linear stability analysis. Existence and

uniqueness of solutions. Impossibility of oscillations in one dimension, Potentials, Solving on the computer.harmonic Flows on the Circle : Possibility of oscillations, Superconducting Josephson Junction, Equivalent circuit and damped, driven pendulum analogue. (c) Bifurcations in one dimensional systems and their classifications. Imperfect bifurcations and catastrophes. [13 lecture hours]
Two-Dimensional Flows: (a) Linear Systems and classification. Nonlinear systems: linearization and Jacobian matrix, analysis in polar coordinates. Conservative systems, reversible systems. (b)Lyapunov function, gradient systems, Dulac criterion, limit cycle, Poincare-Bendixson theorem, Lienard systems. Analysis of two widely separated time-scales. (c) Bifurcations in two dimensions: Hopf Bifurcation-super and sub-critical. [12 lecture hours]
Chaos I: One dimensional map : Stability, Liapunov exponent, chaos; Logistic map : period-doubling route to chaos, Renormalisation arguments.[10 lecture hours]
Chaos II: Fractals : examples, similarity dimension and box dimension; Rayleigh-Benard convection : basic equations, Boussinesq aproximation; Lorenz map : Stability of fixed points and appearence of strange attractors; Baker's map; Henon map : relation with periodically kicked rotator, stability of fixed points and appearence of strange attractors.[12 lecture hours]
Quantum Chaos: Elementary ideas of quantum chaos. [3 lecture hours]
Suggested Texts:

- S. H. Strogatz, Nonlinear Dynamics and Chaos (Westview Press, Indian Edition by Levant Books, Kolkata 2007),
- R.L. Devaney, An Introduction to Chaotic Dynamical Systems (Benjamin-Cummings, 1986, Second Edition),
- D.W. Jordan and P. Smith, Nonlinear Ordinary Differential Equations (Oxford University Press, 2007, 4th Edition),
- G.L. Baker and J.P. Gollub, Chaotic Dynamics - An Introduction (Cambridge University Press, 1996, Second Edition),
- E. Ott, Chaos in Dynamica Systems (Cambridge University Press, 2002, Second Edition),
- H.G. Schuster and W. Just, Deterministic Chaos - An Introduction (Wiley-VCH, 2005, 4th Edition).

Elective M 352: Differentials Forms:
Syllabus
Highlights in multivariable calculus, covered in the course Analysis II, are the integral theorems of Stokes and Gauss which may be viewed as two- and three-dimensional versions of the fundamental theorem of calculus. However, the definitions of curl and divergence appear rather ad hoc, and it would be desirable to obtain these differential opera- tors in a natural way from a general concept. The goal of this course is to build exterior calculus, leading up to

a general Stokes theorem about integration of differential forms on manifolds which contains the integral theorems mentioned above as special cases. This course starts with defining differential one-forms on R^n. Then we introduce the wedge product, thus producing k-forms. We generalize the concepts of differentiation and integration from calculus for differential forms and investigate how a change of variables affects integration. We will also meet the important Poincar e lemma on the relation of closed and ex- act differential forms. All concepts are then generalized to differential forms on manifolds. Finally, we derive the generalized Stokes theorem. Emphasis is being put on geometric visualization in order to gain a good intuition for the concepts described above. Our main textbook will be "A Visual Introduction to Differential Forms and Calculus on Manifolds" (Springer International Publishing, harmonic2019) by J.P. Fortney which is particularly strong on the intuitive-geometric side.
Suggested Texts:

- J.P. Fortney, "A visual introduction to differential forms and caluculus", Birkhaeuser, 468p. (1918),
- M. Spivak, "Calculus on Manifolds. A modern approach to classical theorems of advanced calculus", New York-Amsterdam: W.A. Benjamin, Inc. 1965, 144 p. (1965).

Project M400: At the discretion of faculty, a student might be allowed to carry out a one semester project on an advanced topic. The project should slowly begin in Sem 3 itself and carried out through the 4th semester in lieu of one elective. The project is meant to develop both oral and written presentation skills of the student and wherever possible, inculcate a taste for research in the student

Research Methodology M450: This course is a compulsory requirement for all PhD students. They will be tested in terms of their ability to use software for literature survey (MathSciNet) and prepare latex documents.

Special Topics M 500: Under special circumstances a special topic course might be offered by a faculty member. In such a case, the course content must be approved by the Board of Studies.

- M 501: Topics in Topology
- M 502: Topics in Geometry
- M 503: Topics in Analysis
- M 504: Topics in Dynamics

- M 505: Topics in Algebra
- M 506: Topic s in Analytic Number Theory
- M 507: Topic s in Algebraic Number Theory
- M 508: Topic s in Combinatorial Number Theory
- Seminar M 601: Topology seminar
- Seminar M 602: Analysis seminar
- Seminar M 603: Algebra seminar
- Seminar M604: Number Theory Seminar
- Seminar M605: Discrete Mathematics Seminar
- Research M 700