Arindam Banerjee

Research Statement

September 2014

Department of Mathematics
University of Virginia
Charlottesville, VA

Office: 221 Kerchof Hall
Phone: (785) 218-2568
Email: ab4cb@virginia.edu
http://www.math.virginia.edu/~ab4cb/

1. INTRODUCTION

My current area of research is in the area of commutative algebra and its manifestations in combinatorics and algebraic geometry. Commutative algebra is the study of the commutative rings (e.g. polynomial rings over fields and their quotients) and the modules over these rings. The geometry of the zero sets of system of polynomials gives rise to the field of algebraic geometry while commutative algebra studies the underlying algebraic structures. As expected these two fields are very intimately related and many geometric properties of the zero sets can be measured by the algebraic properties of related commutative rings; for example the geometric notion of smoothness is equivalent to the algebraic notion of regularity (for detailed discussion see the Chapter one in [13]). On the other hand in many cases interesting combinatorial objects can be associated with the commutative rings whose algebraic invariants encode various combinatorial properties. Specially in the case of the quotients of polynomial rings by a squarefree monomial ideal two important combinatorial objects can be associated: a simplicial complex and a hypergraph (see [22] and [12] respectively) and one interesting line of research is to build a dictionary between the algebraic properties of the ring and the combinatorics of the hypergraph or the simplicial complex.

The topic of my doctoral dissertation is broadly in the area of homological algebra. Any module over a commutative ring can be successively approximated by free modules giving rise to a free resolution and by injective modules giving rise to an injective resolution of the module. Many invariants like Betti numbers, Bass Numbers, Castelnuovo-Mumford regularity, projective dimension, injective dimension etc. can be associated with these resolutions that measure "shape" and "size" if interpreted geometrically. I'm interested in building a dictionary between the values of the invariants and the qualitative combinatorial properties of the underlying graphs (see [1], [2], [3], [5], [8], [10], [11], [12], [14], [16], [20], [23], [24], [25]). One important theme of my research in this area is the study of the Castelnuovo-Mumford regularity of the ideals associated to the finite simple graphs. I prove that under various conditions on the graph the regularity of all higher powers of its edge ideal (the squarefree quadratic monomial ideal generated by its edges) has the minimum possible regularity, i.e. their minimal free resolutions are linear. Recently I've also proved that under various conditions on the graph many of its path ideals (the t-path ideal of a graph is the squarefree monomial ideal generated by all its t-paths) have linear resolutions. Another theme of my doctoral research is how to "classify" the quotient rings that are Cohen-Macaulay. One of my result in this area gives a new characterization of the bipartite edge ideal case and my research is mostly driven towards generalizing it in the case of the multipartite hypergraphs.

More recently I've focused on studying two other areas. First of them is the study of the bounds on the regularity and the projective dimension of the ideals generated by homogeneous polynomials and their (in)dependence on the number of variables (More precisely a question asked by Mike Stillman, see the last section for details). Second of them is the study of the so called local cohomology modules. After their introduction by Grothendieck (see [9]), the local cohomology modules have been the focus of research of many mathematicians. The vanishing of the local cohomology seems to be connected to almost every part of commutative algebra and a broad range of topics from algebraic geometry. One particular problem that interests me is how the structure of the spectrum of the underlying ring is reflected in the vanishing of the local cohomology modules (see [15], [17]). I wish to explore this area with the hope that other than being an interesting area of research on its own right this will also enrich my mathematical knowledge.

The subsequent sections are devoted to my research projects and to my future research plans.

2. Homological Algebra of ideals related to simple graphs

Let M be a standard graded module over a polynomial ring S. It is known that M can be successively approximated by free modules. Formally speaking there exists an exact sequence of minimal possible length called minimal free resolution of M:

$$0 \longrightarrow \mathbb{F}_p \xrightarrow{.d_p} \mathbb{F}_{p-1} \cdots \xrightarrow{.d_2} \mathbb{F}_1 \xrightarrow{.d_1} \mathbb{F}_0 \xrightarrow{.d_0} M \longrightarrow 0$$

Where $\mathbb{F}_i = \bigoplus S(-j)^{\beta_{ij}}$. Here $S(-j)$ is the free module with one generator which has degree j. These β_{ij}s are called the Betti numbers of M. Two very important invariants that are defined in terms of these numbers are Castelnuovo-Mumford regularity or simply regularity and projective dimension, denoted by $\mathrm{reg}(M)$ and $\mathrm{pd}(M)$ respectively:

$$\mathrm{reg}(M) = \max\{j - i | \beta_{ij} \neq 0\}$$

$$\mathrm{pd}(M) = \max\{i | \text{there is a } j, \beta_{ij} \neq 0\}$$

If all the d_is are generated by linear polynomials then M is said to have a *linear minimal free resolution.* The linear minimal free resolution is the case of minimum possible regularity.

There are various monomial ideals that are associated to graphs. Among these, the edge ideals and the path ideals along with their powers and colons have been one of the big focus of my research.

For any graph G with set the of vertices $x_1, ..., x_n$ let S be the polynomial ring on $x_1, ..., x_n$ over any field. The edge ideal $I(G)$ and the t-path ideal $I_t(G)$ is defined as follows:

$$I(G) = (x_i x_j | x_i x_j \text{ is an edge in } G)$$

$$I_t(G) = (x_{i_1} .... x_{i_t} | x_{i_1} .... x_{i_t} \text{ is a } t\text{-path in } G)$$

My research on homological algebra of the monomial ideals related to the graphs can be broadly divided into two projects: 1. Finding the upper bounds of regularity and 2. classifying the classes of ideals whose quotients are Cohen-Macaulay.

2.1. **Regularity Of Powers Of Edge Ideals And Path Ideals.**

With the advent of the computer algebra systems the explicit computations of the minimal free resolutions of the modules over the polynomial rings became easily accessible to mathematicians and questions regarding the explicit values of several homological invariants have started to become popular. Castelnuovo-Mumford or simply regularity is one such invariant of the graded modules over the polynomial rings. In simple terms it is a measure of the complexity in the sense that the ideals generated in lower degrees are less complex than those generated in higher degrees.

Under this theme the study of regularity of the squarefree monomial ideals were made popular by various people. The cases where these ideals have linear minimal free resolutions, i.e. the maps in their minimal free resolutions are generated by linear polynomials, became of particular interest. Any squarefree quadratic monomial ideal is naturally associated with a finite simple graph as its edge ideal and the following result by Fröberg (see [8]) indicated the close connection between the combinatorial structure of the graph and the algebraic properties of the ideal:

Theorem 2.1. *(Fröberg) Let G be a simple graph with edge ideal $I(G)$. The minimal free resolution of $I(G)$ is linear if and only if the complement graph G^c is chordal that is every cycle of length greater than three has a chord.*

As the minimal free resolutions of the powers of an ideal can be read off from the resolution of the Rees algebra of the ideal, one expects that the combinatorial structure of the graph to have control over the resolutions of higher powers of the edge ideal too. The following result by Herzog, Hibi and Zheng (see [14]) provides a strong evidence in favor of that:

Theorem 2.2. *(Herzog, Hibi, Zheng) If $I(G)$ has linear minimal free resolution (or equivalently G^c is chordal) then so does every power of $I(G)$.*

It is well known that the linear minimal free resolution is the case of minimum Castelnuovo-Mumford regularity. In this case the linear minimal free resolution for $I(G)^k$ is the case where $\mathrm{reg}(I(G)^k) = 2k$. Very little is known about the interplay between the combinatorics of the graph and the algebra of $I(G)$ when the regularity is not minimal but one looks for results of the similar flavor in those cases also.

One particular case of interest is to study under what assumptions on the graph the higher powers of $I(G)$ have linear minimal free resolution as it is known that the converse of the previous theorem is not true, for example the edge ideal of a five cycle does not have a linear minimal free resolution but all its powers do. It is also known that if any power of $I(G)$ has a linear minimal free resolution then G^c does not have any induced 4-cycle (see [25]). In the light of these and Macaulay 2 calculations the following open question was raised by Nevo and Peeva (see [25]):

Question 2.3. *(Nevo, Peeva) If G has no 4-cycle in its complement and reg$(I(G)) \leq 3$ then is it true that all higher powers of $I(G)$ have linear minimal free resolution?*

It is realized by several people that the short exact sequences coupled with close study of the colon ideals can be very effective in finding bounds for the regularities of the monomial ideals. Motivated by the work of Dao, Huneke and Schweig in [3], which is of the same spirit, the short exact sequences and the colon ideals related to the powers of the edge ideals were closely studied by my self and the following property of the regularities of the powers of edge ideals was proved (see [2]):

Theorem 2.4. *(-) For any finite simple graph G and any $s \geq 1$, let the set of minimal monomial generators of $I(G)^s$ be $\{m_1, ..., m_k\}$, then $((I(G))^{s+1} : m_l)$ is a quadratic monomial ideal and*

$$\operatorname{reg}(I(G)^{s+1}) \leq \max\{\operatorname{reg}(I(G)^{s+1} : m_l) + 2s, 1 \leq l \leq k, \operatorname{reg}(I(G)^s)\}$$

Using this result I proved a series of results regarding the regularities of the powers of the edge ideal. My main two results in [2] are summarized as the following theorem:

Theorem 2.5. *(-) If G is s graph with no four cycle in complement such that reg$(I(G)) = r$ then reg$(I(G)^s) \leq 2s + r - 1$. If further G is cricket free (see [2] for definition) then reg$(I(G)) \leq 3$ and for all $s \geq 2$, reg$(I(G))^s = 2s$.*

This theorem partially answers the aforesaid question and at present more work in this direction is being pursued by me.

Recently I've explored the use of Theorem 3.4 in the case of powers of the bipartite edge ideals. Oscar Fernendez-Ramos and Phillippe Gimenez have recently proved a characterization of the regularity 3 bipartite edge ideals. Using this characterization and the Theorem 3.4, Ali Alilooee and I have shown the following (see [1]):

Theorem 2.6. *(Alilooee,-) If $I(G)$ is a bipartite edge ideal of regularity 3 then the regularity of $I(G)^s$ is exactly $2s + 1$ for all s.*

I've also got interested in path ideals. I've been interested in the cases where the path ideal have a linear minimal free resolution and whether its regularity has any relations with the corresponding edge ideal. In a recent work I have addressed these questions and achieved various results. My main result is the following:

Theorem 2.7. *(-) If G is a claw free (see [2] for definition) finite simple graph such that G^c does not have any induced four cycle then every non zero path ideal have linear minimal free resolution.*

Further exploration in these direction is ongoing. Along with the study related to the Question 2.3, right now my research in this direction is focused on three questions. The first one was posed by Dao, Huneke and Schweig in [2]:

Question 2.8. *Let G be a graph without any induced four cycle in its complement. Can reg$(I(G))$ be arbitrarily large?*

It seems to me that the solution to this problem is closely related to a "description" (algebraic/combinatorial) of the set of all variables x for which $\operatorname{reg}(I) = \operatorname{reg}(I, x)$ for the gap free graphs. Till now no such description exists. Either via such a description or by some other means if one can at least estimate the size of that set, it is expected that some progresses towards this question will come.

The second question is motivated by the Theorem 3.4:

Question 2.9. *Under what combinatorial condition on G and for which $s \geq 2$, reg$((I(G))^s : m) \leq$ reg$((I(G))$ for every minimal monomial generator m of $I(G)^{s-1}$.*

In the proof of the cricket free part of the Theorem 3.4 and the proof of the Theorem 3.5, a crucial point was to show that $\mathrm{reg}(I(G)^s : m) \leq \mathrm{reg}(I(G))$ for every minimal monomial generator m of $I(G)^{s-1}$ using the combinatorial description of G. Further exploration of this type is expected to give similar theorems and also shed more light to the Question 3.1.

The third one is related to the path ideals. Not much is known about the regularities of the powers of the path ideals and their relations with the regularities of the path ideals themselves and the regularity of the edge ideal. Macaulay 2 examples show that in many cases if the edge ideal has linear resolution then all the path ideals and all of their powers have linear resolutions. However the number of examples computed is not high enough to conjecture that this is always true. The question in its most general form is the following:

Question 2.10. *Depending on the regularity of edge ideal and path ideals can one find an optimum upper bound for regularity of a fixed power of a fixed path ideal?*

One crucial tool that can be used for the powers of the edge ideals and not readily available for the powers of the path ideals is the Theorem 3.4. One possible approach to attack this problem is to prove a similar theorem for the path ideals (even for a restricted family!). Another approach can be to use the Theorem 3.4 itself judiciously. As the combinatorics of the path ideals is intimately connected to the edge ideals, it might be possible to connect the powers of the path ideals and the related colon ideals to the powers of the edge ideals.

Answers to these three questions are expected to lead towards further results regarding the regularities of the powers of the edge ideals and in the course of finding them one is expected to discover some new techniques too.

2.2. **Cohen-Macaulay Monomial Ideals.** The Cohen-Macaulay rings and modules have many beautiful homological and geometric properties. It has always interested algebraists to find new classes of rings and modules that are Cohen-Macaulay. In the case of squarefree monomial ideals, there is a hypergraph and a simplicial complex associated with it in a natural way, and studies have been done to interlink the Cohen-Macaulayness of the quotient ring and the combinatorics of the hypergraph and the simplicial complex.

The relation between the combinatorics of the simplcial complex and the Cohen Macaulayness of the monomial ideal is connected via the Alexander Duality and has been explored extensively (see [22]) and well-understood. The interplay between the combinatorics of the hypergraph (combinatorics of graph) and the Cohen -Macaulaness of the monomial ideal (edge ideals) is comparatively less well-understood, although there have been some works in this direction too . In the case of the edge ideals one is interested to characterizing the Cohen-Macauleyness in terms of the combinatorics of the graph. The problem in its full generality is wide open however some special cases are known. The following result by Herzog and Hibi is such a characterization in the case of bipartite graphs:

Theorem 2.11. *(Herzog, Hibi) Let G be a bipartite graph with bipartition $\{x_1, ..., x_n\}$ and $\{y_1, ..., y_n\}$. Then $I(G)$ is Cohen Macaulay if and only if there exists an enumeration of x's and y's with the following three properties:*
a. $x_i y_i \in I(G)$.
b. $x_i y_j \in I(G) \implies i \leq j$
c. $x_i y_j \in I(G), x_j y_k \in I(G) \implies x_i y_k \in I(G)$.

Recently I have proved the following new characterization of the Cohen Macaulay bipartite edge ideals:

Theorem 2.12. *(-) Let G be a finite simple bipartite graph whose partitions are of size n and its edge ideal is $I(G)$. Then $I(G)$ is Cohen-Macaulay if and only if there exists exactly n edges e of $I(G)$ such that $(I(G)^2 : e)$ is Cohen-Macaulay and all those edges are disjoint.*

the case of higher degree monomial ideals not much is known about the relation between the combinatorics of the hypergraph and the Cohen-Macaulayness of the ideal. Some partial results have been achieved. One case that seems to be interesting and also natural in the light the characterization of the bipartite edge ideals is the case of the uniform multipartite hypergraph.

Question 2.13. *Is there a generalization of either Theorem 3.11 or 3.12 for t-uniform, t-partite hypergraphs?*

Some partial results (a sufficient condition for the 3-uniform, 3-partite hypergraph) has been achieved by me and more research is ongoing.

Another question which seems to be of interest is the Cohen-Macauleyness of the path ideals and and its relation with the edge ideals. In general neither of them implies the other which is explained in the following example:

Example 2.14. *Let $S = K[x, y, z]$. If $I = (xy, xz)$ then it is an edge ideal which is not Cohen-Macaulay but the corresponding three path ideal $J = (xyz)$ is definitely Cohen-Macaulay. On the other hand let $S' = K[x, y, z, w]$. If $I' = (xy, xw, zw)$ then it is an edge ideal which is Cohen-Macaulay but the corresponding 3-path ideal (xyw, xzw) is not Cohen-Macaulay.*

However it seems interesting to find classes of graphs where there is a relation between the two.

Question 2.15. *For which classes of graphs, Cohen-Macaulayness of edge ideals imply Cohen-Macaulayness of path ideals or vice versa?*

Work in this direction is ongoing and success is expected to come along with further directions of research.

3. Local Cohomology and Lyubeznik Numbers

For any module M over a commutative ring R and an ideal $I = (f_1, ..., f_k)$ of R we denote the i-th local cohomology modules by $H^i_I(M)$ which is defined in the following way:
It is the i-th cohomology of the following complex:

$$0 \cdots M \longrightarrow \oplus M_{f_i} \longrightarrow \oplus M_{f_i f_j} \cdots \longrightarrow M_{f_1 \dots f_k} \longrightarrow 0$$

Here the differentials are defined by localization.

After its introduction by Grothendieck, the local cohomology has been the focus of the research of many commutative algebraists and algebraic geometers over decades. Its close connections with several areas of mathematics including derived functors, theory of unique factorization domains, sheaf cohomology and its several beautiful properties like local duality makes it both a very useful tool and a very interesting topic of research on its own. Until now, my research in the local cohomology has two foci: the study of the application of the local cohomology in general and the local duality in particular in studying the homological invariants and the properties of the Lyubeznik Numbers.

3.1. Application of Local Duality.

Eisenbud, Huneke and Ulrich in [6] studied the application of the local cohomology in the study of the regularity Tor modules and proved the following result about the regularity of Tor using the local duality:

Theorem 3.1. *(Eisenbud, Huneke, Ulrich) Let A and B be finitely generated standard graded modules over a polynomial ring S in n variables over a field $\mathbb{K}$ with dim $Tor_1(A, B) \leq 1$ and let j, k be integers. Let $p \leq$ codim A and $q \leq$ codim B and $p + q = n - j + k$ then,*

$$reg(H^j_m(Tor_k(A, B))) \leq reg(Tor_p(A, \mathbb{K})) + reg(Tor_q(B, \mathbb{K}) - n$$

Using this result they proved several results on the regularity and on the syzygies. I've been interested in generalizing the Theorem 4.1 in various directions and find applications. In particular the following generalization in the multigraded case has been proved by me:

Theorem 3.2. *(-) Let A and B be finitely generated multigraded modules over a polynomial ring $S = \mathbb{K}[x_1....x_n]$ with dim $Tor_1(A, B) \leq 1$ and let j, k be integers. Let $p \leq$ codim A and $q \leq$ codim B and $p + q = n - j + k$. If $H^j_m(Tor_k(A, B))_\sigma \neq 0$ for some monomial σ then there exists monomials α and β with $Tor_p(A, \mathbb{K})_\alpha \neq 0$ and $Tor_q(B, \mathbb{K}_\beta \neq 0$ where $x_1....x_n\sigma$ is divisible by $\alpha\beta$.*

Further work in this direction specially to find a way to avoid or weaken the condition on the dimension (even for a restricted class of modules) is ongoing. In particular in the monomial case it will be interesting to find a combinatorial condition that replaces the condition on the dimension. These are summarized in the form of a question below:

Question 3.3. *Can the dimension condition of Theorem 4.1 be replaced by a weaker condition even for a restricted class of modules? In the case where the modules are quotients by monomial ideals can one replace the dimension condition by a combinatorial condition?*

One possible approach towards this, seems to study the spectral sequence used in the proof of Theorem 4.1 closely (see [6]). Its collapse in E^2 page seems to be extremely important for the proof. In that page most of the modules vanish because of the condition on the dimension. Whether under a weaker condition one can achieve at least the vanishing of the maps (instead of asking for the modules themselves to be zero) is not fully understood. One can try to look for some combinatorial conditions that facilitate this. Another possible approach is to try to replace that spectral sequence under some conditions by several short exact sequences. This will open up doors to several possible generalizations.

3.2. **Lyubeznik Numbers.** The other direction related to the local cohomology that I've worked in is the properties of the Lyubeznik numbers. In 1993 Lyubeznik introduced a family of invariants for a local ring containing a field, R, today called the Lyubeznik numbers and denoted by $\lambda_{i,j}(R)$. These numbers have been shown to have multiple connections; for instance, they relate to the singular and the ètale cohomology, to the Hochster-Huneke graph (see [15]), and to the projective varieties. These connections have motivated multiple generalizations; for instance, for mixed characteristic rings , and rings of equal-characteristics from a differential perspective.

In a recent joint work Núñez-Betancourt, Yanagawa and I have studied the behavior of these invariants. The first result obtained in this context is that the Lyubeznik numbers are bounded globally over the rings of positive characteristic.

Theorem 3.4. *(-, Núñez-Betancourt, Yanagawa) Let R be an F-finite ring which is a quotient of a regular Noetherian ring of finite dimension and positive characteristic $p > 0$. Then, there exists a positive integer B such that*

$$\lambda_{i,j}(R_Q) \leq B$$

for every $i, j \in \mathbb{N}$ and $Q \in \operatorname{Spec}(R)$.

The previous claim was proved for algebras finitely generated over a field of characteristic zero or an algebraically closed field of prime characteristic by Puthenpurakal. We pointed out that his result dealt only with the localization at the maximal ideals.

Unfortunately, the Lyubeznik numbers do not behave much better than what it is stated in the previous theorem. We show an example of a Stanley-Reisner ring whose highest Lyubeznik number could either decrease or increase under localization.

An operation related with the localization of the Stanley-Reisner rings is the polarization. Given a monomial ideal I, not necessarily radical, in a polynomial ring S, we consider the polarization ideal $\widetilde{I}$ in the polarization ring $\widetilde{S}$. We compare the Lyubeznik numbers at the maximal homogeneous ideal of $S/\sqrt{I}$ and $\widetilde{S}/\widetilde{I}$.

Theorem 3.5. *(-, Núñez-Betancourt, Yanagawa) Let $S = K[x_1, \dots, x_n]$ be a polynomial ring, $\mathfrak{m} = (x_1, \dots, x_n)$ and $I \subset S$ be a monomial ring. Let $\widetilde{I}$ denote the polarization of I, and $\widetilde{S} = K[x_{r,s}]$ denote the polarization ring. Let $\eta = (x_{i,j})$, and $h = \dim(\widetilde{S}/\widetilde{I}) - \dim(S/I)$. Then,*

$$\lambda_{i-h,j-h}\left(S_{\mathfrak{m}}/IS_{\mathfrak{m}}\right) = \lambda_{i,j}\left(\widetilde{S}_{\eta}/\widetilde{I}\widetilde{S}_{\eta}\right).$$

for every $i, j \in \mathbb{N}$.

This result gives surprisingly different behavior of Lyubeznik numbers under the localization and the polarization. We also study the behavior of the the generalized Lyubeznik numbers, $\lambda_i^0(R)$, for rings of equal characteristic under the localization and the polarization for the Stanley-Reisner rings. As a consequence of our methods, we observe similar behavior for the finer invariants given by the multiplicities of the characteristics cycles.

One question regarding the Lyubeznik numbers that I wish to study in the near future is the following:

Question 3.6. *Is there a relation between Lyubeznik numbers of edge ideals and Lyubeznik numbers of related path ideals?*

For the Betti numbers people have studied the relation between the Betti numbers of the edge ideals and their powers. Those questions of course are irrelevant in case of the Lyubeznik numbers as they are invariant up to the radical; but one can ask the similar question about the relations between the Lyubeznik numbers of the edge ideal and the path ideals and study whether in some cases a nice theory can be developed.

Not much is known about this question. However it seems that better understanding of the relationship between the associated primes of the edge ideals and that of the path ideals might shed some light.

4. Research Plans For Future

Along with the ongoing research on the aforesaid three themes (i.e. the questions mentioned in the previous sections) right now I'm studying the research papers and the other available literature in two other topics on which I wish to pursue research in the near future. First of them is a question asked by Mike Stillman on the dependence of the projective dimension on the number of variables of the underlying polynomial ring that has been studied by many researchers in recent years and the second one is the problem of the vanishing of the local cohomology.

4.1. **Stillman's question on Projective Dimension.** The following question was asked by Mike Stillman:

Question 4.1. *Let $f_1, ..., f_k$ be k homogeneous polynomials of degrees $d_1, ..., d_k$ in a polynomial ring in n variables over any field K. Is there an upper bound for the projective dimension of $I = (f_1, ..., f_k)$ depending only on $d_1, ..., d_k$ independent of n?*

In [7] the case of the three cubics has been solved by Bahman Engheta. In [18] and [19], Huneke, Mantero, Seceleanu and McCullough have studied the quadric case and recently Hochster and Annayan have announced that they have a proof for all the cases up to degree four. The general case is very much open and even in the known cases the bounds are far from being sharp. The methods used in each paper mentioned above are quite different from the others. Possibility of pursuing research in various directions using various tools both personally and collaborating with others makes this area look like an extremely interesting field for future research.

4.2. **Vanishing of Local Cohomology.** Lastly I've started studying the available works related to the vanishing of the local cohomology modules. This is an area which has been studied by researchers extensively. To mention a few [15],[17]. This area has been a melting pot of tools and techniques from every part of commutative algebra making this a ripe field to explore. One direction in which I'm particularly interested in pursuing research is to study how the structure of the spectrum of the underlying ring influences the local cohomology.

Apart from this specific research plan, I'm interested in studying various questions arising from local cohomology, homological algebra, structure of spectrum of rings, topological properties of simplicial complexes arising from algebra. My long term goal is to take up some of these studies.

References

[1] A. Alilooee, A. Banerjee, Powers of edge ideals of regularity three bipartite graphs, arXiv:1408.2557.

[2] A. Banerjee, Regularity of powers of edge ideals, accepted for publication in J. Algebraic Combinatorics.

[3] H. Dao, C.Huneke J. Schweig, Bounds on the regularity and projective dimension of ideals associated to graphs, J. Algebraic Combin. **38** (2013), 37–55.

[4] D. Eisenbud, Commutative algebra. With a view towards algebraic geometry. Graduate Texts in Mathematics, Springer-Verlag, New York, **150** (1995).

[5] D. Eisenbud, M. Green, K. Hulek, and S. Popescu, Restricting linear syzygies: algebra and geometry, Compos. Math. **141** (2005), 1460–1478.

[6] D. Eisenbud, C. Huneke, B. Ulrich, The Regularity of Tor and Graded Betti Numbers, American Journal of Mathematics, **128**, No. 3, (2006), 573-605.
[7] B. Engheta, A bound on the projective dimension of three cubics, Journal of Symbolic Computation **45**, (2010), 60-73.
[8] R. Fröberg, On Stanley-Reisner rings, Topics in Algebra, Banach Center Publications, **26** (2) (1990), 57–70.
[9] A. Grothendieck (Notes by R. Hartshorne), Local Cohomology, Springer Lecture Notes in Math., **41**, Springer-Verlag, (1966).
[10] H. Tài Hà and A. Van Tuyl, Splittable ideals and the resolutions of monomial ideals, J. Algebra **309** (2007), 405–425.
[11] H. Tai Ha and A. Van Tuyl, Resolution of square-free monomial ideals via facet ideals: a survey, Contemporary Mathematics **448** (2007), 91–117.
[12] H. Tai Ha and A. Van Tuyl, Monomial ideals, edge ideals of hypergraphs and their graded Betti numbers, Journal of Algebraic Combinatorics **27** (2008), pp 215-245.
[13] R. Hartshorne, Algebraic Geometry, Graduate Texts in Mathematics, Springer-Verlag, **52**, (1977).
[14] J.Herzog, T.Hibi, X.Zheng: Monomial ideals whose powers have a linear resolution, Math. scand. **95** (2004), 23-32.
[15] M. Hochster, H. Huneke, Indecomposable canonical modules and connectedness, Contemporary Math., **159** (1994), 197-208.
[16] A.H.Hoefel, G.Whieldon: Linear Quotients of square of the edge ideal of the anticycle, arXiv:1106.2348v2.
[17] C. Huneke, G. Lyubeznik, On the vanishing of local cohomology modules, Inventiones Math., **102** (1990), 73-93.
[18] C. Huneke, P. Mantero, J. McCullough, A. Seceleanu, Ideals Generated by 4 Quadric Polynomials, arXiv:1403.6334.
[19] C. Huneke, P. Mantero, J. McCullough, A. Seceleanu, The projective dimension of codimension two algebra presented by quadrics, arXiv:1304.0745.
[20] M. Kummini, Regularity, depth and arithmetic rank of bipartite edge ideals, J. Algebraic Combin. **30** (2009), 429–445.
[21] M. Kummini, Homological invariants of monomial and binomial ideals, thesis, University of Kansas (2008).
[22] E. Miller and B. Sturmfels, Combinatorial commutative algebra, Graduate Texts in Mathematics, Springer-Verlag, New York, **227** (2005).
[23] S. Morey and R. Villarreal, Edge ideals: algebraic and combinatorial properties, Progress in Commutative Algebra: Ring Theory, Homology and Decomposition, de Gruyter, Berlin (2012), 85–126.
[24] E. Nevo, Regularity of edge ideals of C_4-free graphs via the topology of the lcm-lattice, J. Combin. Theory Ser. A **118** (2011), 491–501.
[25] E. Nevo and I. Peeva, C_4-free edge ideals J Algebraic Combin. **37** (2013), 243–248.

Department of Mathematics, University of Virginia, Charlottesville, VA, USA
E-mail address: `ab4cb@virginia.edu`